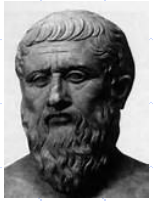
Formal Semantics in Modern Type Theories

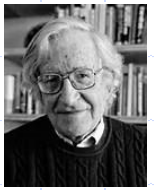
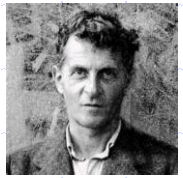
(MTT-semantics is both model/proof-theoretic)

Zhaohui Luo
Royal Holloway
University of London

Natural Language Semantics

- Semantics – study of meaning (communicate = convey meaning)
- Various kinds of theories of meaning
  - Meaning is reference ("referential theory")
    - Word meanings are things (abstract/concrete) in the world.
    - c.f., Plato, …
  - Meaning is concept ("internalist theory")
    - Word meanings are ideas in the mind.
    - c.f., Aristotle, …, Chomsky.
  - Meaning is use ("use theory")
    - Word meanings are understood by their uses.
    - c.f., Wittgenstein, …, Dummett.




Type-Theoretical Semantics

- Montague Semantics (MG)
  - R. Montague (1930–1971)
  - Dominating in linguistic semantics since 1970s
  - Set-theoretic, using simple type theory as intermediate
  - Types ("single-sorted"): e, t, e→t, …

- MTT-semantics: formal semantics in modern type theories
  - Examples of MTTs:
    - Martin-Löf's TT: predicative; non-standard FOL
    - pCIC (Coq) & UTT (Luo 1994): impredicative; HOL
  - Ranta (1994): formal semantics in Martin-Löf's type theory
  - Recent development ➔ full-scale alternative to MG
  - Many types ("many-sorted"): Table, ∑(Man, handsome), Phy•Info

- Recent development on rich typing in NL semantics
  - MTT-semantics is one of these developments.
  - Chatzikyriakidis and Luo (eds.) Modern Perspectives in Type Theoretical Sem. Springer, 2017. (Collection on rich typing)
  - Chatzikyriakidis and Luo. Formal Semantics in Modern Type Theories. Wiley/ISTE. (Monograph on MTT-sem, to appear)
- Advantages of MTT-semantics, including
  - Both model-theoretic & proof-theoretic – offering a new perspective not available before.
  - Today: focus on this after introducing MTT-semantics.

MTT-semantics compared with Montague sem.

	Example	Montague semantics	Semantics in MTTs
CN	man, human	$[\![man]\!],\ [\![human]\!] : e \to t$	$[\![man]\!],\ [\![human]\!] : Type$
IV	talk	$[\![talk]\!] : e \to t$	$[\![talk]\!] : [\![human]\!] \to Prop$
Adj	handsome	$[\![handsome]\!] : (e \to t) \to (e \to t)$	$[\![handsome]\!] : [\![man]\!] \to Prop$
MCN	handsome man	$[\![handsome]\!]([\![man]\!])$	$\sum[m : [\![man]\!],\ h : [\![handsome]\!](m)]$
S	A man talks	$\exists m : e.\ [\![man]\!](m) \& [\![talk]\!](m)$	$\exists m : [\![man]\!].\ [\![talk]\!](m)$

E.g., in MTT-semantics, CNs are types rather than predicates:

(*) John is a man.

- Montague: man(j) where man : e→t
- MTT-sem: j : Man where Man : Type

(#) The table talks. – What about talk(t)?

- Well-typed/false in Montague (talk : e→t & t : e)
- Untypable/meaningless in MTT-sem (talk : Human→Prop & t:Table)
- "selectional restriction": meaningfulness v.s. truth

Modelling Adjective Modifications [CL13, Luo18, XLC18]

Classical classification	**Example**	**Characterisation of** Adj(N)	**MTT-semantics**
intersective	handsome man	N & Adj	$\sum$x:Man.handsome(x)
subsective	large mouse	N (Adj depends on N)	large : ΠA:CN. A→Prop large(mouse) : Mouse→Prop
privative	fake gun	$\neg$N	$G = G_R + G_F$ with $G_R \leq_{inl} G$, $G_F \leq_{inr} G$
non-committal	alleged criminal	nothing implied	$\exists$h:Human. B_h(...)

Note on Subtyping

- Subtyping essential for MTT-semantics
  - Could a "handsome man" talk?
  - Paul talks ➔ talk(p)?

    where talk:Human→Prop and p:[handsome man]
  - talk(p) : Prop, because

    p : [handsome man] = Σ(Man,handsome) $\leq$ Man $\leq$ Human
- Remarks
  - Subtyping is crucial for MTT-semantics.
  - Coercive subtyping [Luo97, XLS12] is adequate for MTTs and we use it in MTT-semantics.

Advanced features in MTT-semantics: examples

- Anaphora analysis
  - MTTs provide alternative mechanisms for proper treatments via Σ-types [Sundholm 1989] (cf, DRTs, dynamic logic, …)
- Linguistic coercions
  - Coercive subtyping provides a promising mechanism [Asher & Luo 2012]
- Copredication
  - Cf, [Pustejovsky 1995, Asher 2011, Retoré et al 2010]
  - Dot-types [Luo 2009, Xue & Luo 2012, Chatzikyriakidis & Luo 2018]
- Several recent developments
  - Dependent event types in event sem. [Luo & Soloviev (WoLLIC17, TYPES19)]
  - Propositional Forms of Judgemental Interpretations [Xue et al (NLCS18)]
  - CNs as Setoids [Chatzikyriakidis & Luo (J of Oslo meeting 2018)]
  - HoTT-logic for MTT-semantics in Martin-Löf's TT (LACompLing18)

MTT-semantics is both model/proof-theoretic

- Model-theoretic semantics (traditional)
  - Meaning as denotation (Tarski, …)
  - Montague: NL → (simple TT) → set theory
- Proof-theoretic semantics
  - Meaning as inferential use (proof/consequence)
  - Gentzen, Prawitz, …, Martin-Löf
  - e.g., Martin-Löf's meaning theory
- MTT-semantics
  - Both model-theoretic and proof-theoretic – in what sense?
  - What does this imply?

Formal semantics in Modern Type Theories (MTT-semantics) is both model-theoretic and proof-theoretic.

- NL → MTT (representational, model-theoretic)
  - MTT as meaning-carrying language with its types representing collections (or "sets") and signatures representing situations
- MTT → meaning theory (inferential roles, proof-theoretic)
  - MTT-judgements, which are semantic representations, can be understood proof-theoretically by means of their inferential roles
- Z. Luo. Formal Semantics in Modern Type Theories: Is It Model-theoretic, Proof-theoretic, or Both? Invited talk at LACL14.

MTT-semantics being model-theoretic

- MTTs offer powerful representations.
- Rich type structure
  - Collections represented by types
  - Eg, CNs and their adjective modifications (see earlier slides)
  - Wide coverage – a major advantage of model-theoretic sem
- Useful contextual mechanisms – signatures
  - Various phenomena in linguistic semantics (eg, coercion & infinity)
  - Situations (incomplete world) represented by signatures (next slide)

MTT-semantics being model-theoretic (cont^{ed})

- Signatures Σ as in (cf, Edin LF [Harper et al 1987])

  $$\Gamma \vdash_{\Sigma} a : A$$

  with $\Sigma = c_1{:}A_1, \ldots, c_n{:}A_n$
- New forms besides c:A [Luo LACL14]

  $$\ldots, c{:}A, \ldots, A \leq_c B, \ldots, c \sim a : A, \ldots$$

  - Subtyping entries (cf, Lungu's PhD thesis 2018)
  - Manifest entries (can be emulated by coercive subtyping)
- *Theorem (conservativity)*

  The extension with new signature entries preserves the meta-theoretic properties for coherent signatures.

MTT-semantics being proof-theoretic

- MTTs are representational with proof-theoretic sem
  - Not available before – cf, use theory of meaning
- MTT-based proof technology
  - Reasoning based on MTT-semantics can be carried out in proof assistants like Coq:
    - pretty straightforward but nice application of proof technology to NL reasoning (not-so-straightforward in the past …)
  - Some Coq codes can be found in:
    - Z. Luo. Contextual analysis of word meanings in type-theoretical semantics. Logical Aspects in Computational Linguistics. 2011.
    - S. Chatzikyriakidis & Z. Luo. NL Inference in Coq. JoLLI 23(4). 2014.
    - S. Chatzikyriakidis & Z. Luo. Proof assistants for NL semantics. LACL 2016.
    - T. Xue et al. Propositional Forms of Judgemental Interpretations. NLCS 2018.

- Why important?
  - Model-theoretic – powerful semantic tools
    - Much richer typing mechanisms for formal semantics
    - Powerful contextual mechanism to model situations
  - Proof-theoretic – practical reasoning on computers
    - Existing proof technology: proof assistants (Coq, Agda, Lego/Plastic, Nuprl)
    - Applications to NL reasoning
  - Leading to both of
    - Wide-range modelling as in model-theoretic semantics
    - Effective inference based on proof-theoretic semantics

Remark: MTT-semantics offers a new perspective – new possibility not available before!