Logical Truths in Constructive Type Theory
(Abstract)

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A constructive type theory such as Martin-Löf's type theory [NPS90], has an internal logic based on the Curry-Howard principle of propositions-as-types. However, as compared with traditional logical systems such as the predicate logic, the internal logic of type theory and the associated notion of logical truth (as opposed to mathematical truth) are much less understood. This is partly because that, based on a rigid interpretation of the propositions-as-types principle as *identifying* propositions and types, as found in Martin-Löf's type theory, to understand the logic is to understand the whole type theory, including the types of mathematical structures.

We argue that, in type theory, there should be a clear distinction between the notion of a logical proposition and that of a type of mathematical structures, and on the basis of this distinction, a further distinction between the notion of (pure) logical truth and that of mathematical truth. Informally, this amounts to an argument for the independence of logic in the sense that the province of logic has a trait of being universal and hence independent of the existence of the entities to be reasoned about by means of the logic. A conceptual identification of logical propositions and types of mathematical structures could be misleading and may make it more difficult to understand the internal logic of type theory.

We consider an informal notion of logical truth in type theory and define a corresponding formal notion in the context of the type theory UTT [Luo94], a natural combination of Martin-Löf's intensional type theory and Coquand-Huet's calculus of constructions [CH88], in which a distinction between logical propositions and data types is already present. We consider a compositional understanding based on the meaning theory of type theory as important in understanding the notion of logical truth, and study the notion of conservative extension as its formal counterpart. (This is related to Prawitz and Dummett's arguments for compositional understanding of languages (cf, [Dum91]). But here, we consider the formal and mathematical language of type theory.) It is hoped that, through this investigation, we can obtain a better understanding of the use of the internal logic of type theory in formalisation of constructive mathematics and, for example, provide criteria as a basis to answer (informally) questions such as 'Is this formalisation adequate'.

References

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