Automatic Cyclic Termination Proofs for Recursive Procedures in Separation Logic

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University College London

TAPAS, Edinburgh, Wednesday 7th September 2016

Automatically Proving Termination: Challenges

```
proc shuffle(x) {
    if x != nil {
        y := *x;
        reverse(y);
        shuffle(y);
    }
}
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recursion

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intermediate procedures

recursion

Automatically Proving Termination: Challenges

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```

heap manipulation

intermediate procedures

recursion

Automatically Proving Termination: Some Solutions

- MUTANT, THOR

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- Mutant, Thor
- Julia, Costa, AProVE

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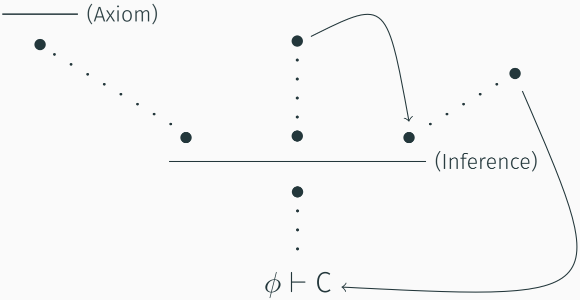
Automatically Proving Termination: Some Solutions

- Mutant, Thor
- Julia, Costa, AProVE
- Dafny
- HipTNT+

Automatically Proving Termination using Cyclic Proof

- Following the approach of Brotherston Et Al. (POPL '08)



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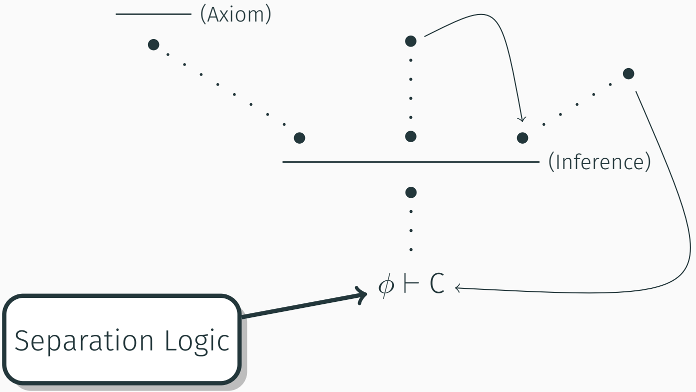

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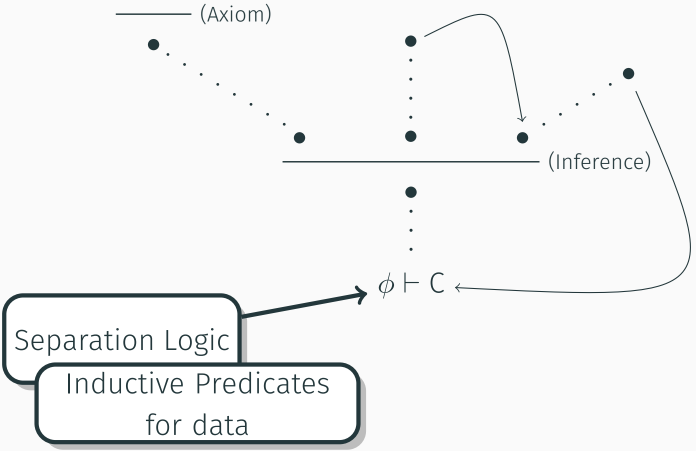

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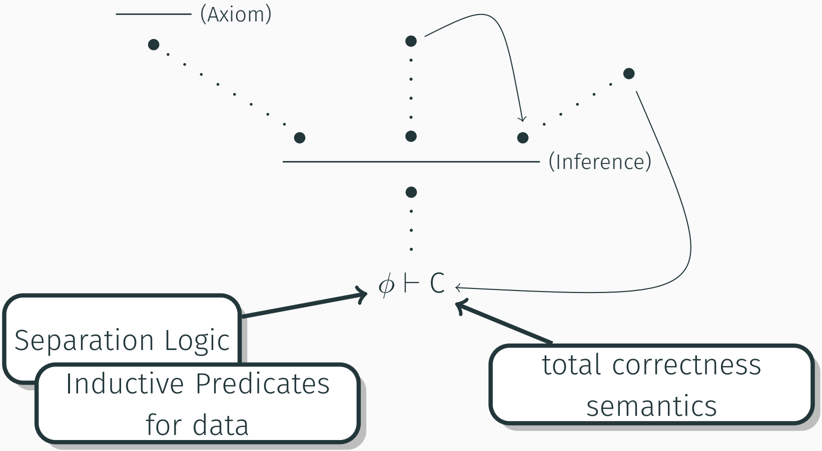

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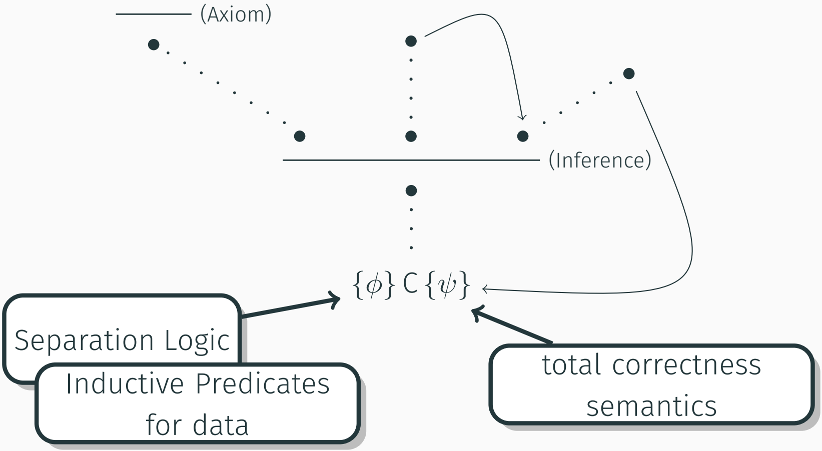

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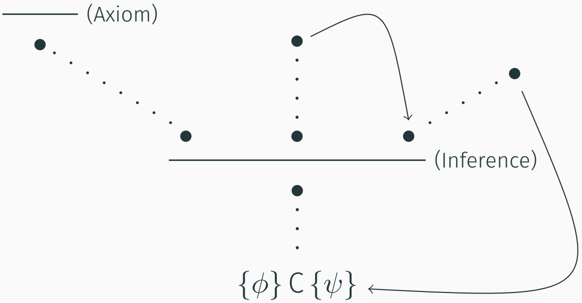

- We use the CYCLIST framework for automation

Advantages of Using Cyclic Proof

- Termination measures extracted automatically

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- Termination measures extracted automatically
- Supports compositional reasoning
- Naturally encapsulates inductive principles

Ingredients of our Approach: Symbolic Execution

$$\text{(free):}\quad \frac{\{\phi\}\ \mathsf{C}\ \{\psi\}}{\{\phi * \mathsf{x} \mapsto \mathsf{y}\}\ \texttt{free(}\mathsf{x}\texttt{);}\ \mathsf{C}\ \{\psi\}}$$

Ingredients of our Approach: Symbolic Execution

$$\text{(free):} \quad \frac{\{\phi\}\,\mathsf{C}\,\{\psi\}}{\{\phi * \mathsf{x} \mapsto \mathsf{y}\}\ \texttt{free(}\mathsf{x}\texttt{);}\ \mathsf{C}\,\{\psi\}}$$

$$\text{(load):} \quad \frac{\{\mathsf{x} = \mathsf{v}[\mathsf{x}'/\mathsf{x}] \wedge (\phi * \mathsf{y} \mapsto \mathsf{v})[\mathsf{x}'/\mathsf{x}]\}\,\mathsf{C}\,\{\psi\}}{\{\phi * \mathsf{y} \mapsto \mathsf{v}\}\ \mathsf{x}\ \texttt{:=}\ \texttt{*}\mathsf{y}\texttt{;}\ \mathsf{C}\,\{\psi\}}\ (\mathsf{x}'\ \text{fresh})$$

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$$\text{(proc):}\quad \frac{\{\phi\}\,\mathsf{C}\,\{\psi\}}{\{\phi\}\;\mathsf{proc}(\vec{\mathsf{x}})\;\{\psi\}}\,(\mathsf{body}(\mathsf{proc}) = \mathsf{C})$$

Ingredients of our Approach: Inductive Predicates

- We support user-defined inductive predicates, e.g.

$$\frac{x = \mathrm{nil} \wedge \mathbf{emp}}{\mathbf{list}(x)} \qquad \frac{x \mapsto y * \mathbf{list}(y)}{\mathbf{list}(x)}$$

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- Explicit approximations used as termination measures, e.g.

$$\{\mathbf{list}_\alpha(\mathsf{x}) * \phi\}\, \mathsf{C}\, \{\psi\}$$

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- A logical rule schema allows case split

$$\frac{\{(x = \mathrm{nil} \land \mathbf{emp}) * \phi\}\, C\, \{\psi\} \quad \{(\beta < \alpha \land x \mapsto y * \mathbf{list}_\beta(x)) * \phi\}\, C\, \{\psi\}}{\{\mathbf{list}_\alpha(x) * \phi\}\, C\, \{\psi\}}$$

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A Cyclic Termination Proof for **shuffle**

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proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }
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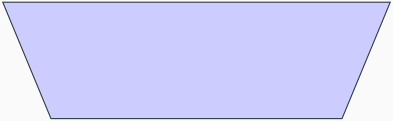

$$\vdots$$

$$\{\mathsf{list}_\beta(y)\}\ \texttt{reverse(y)}\ \{\mathsf{list}_\beta(y)\}$$

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$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \left\{\begin{array}{c}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \operatorname{list}_{\beta}(\mathsf{y})\end{array}\right\} \texttt{rev(y);} \left\{\begin{array}{c}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \operatorname{list}_{\beta}(\mathsf{y})\end{array}\right\}
        \qquad
        \{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \operatorname{list}_{\beta}(\mathsf{y})\}\ \texttt{shuf(y);}\ \{\operatorname{list}_{\alpha}(\mathsf{x})\}
      }{
        \{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \operatorname{list}_{\beta}(\mathsf{y})\}\ \texttt{rev(y);}\ \ldots\ \{\operatorname{list}_{\alpha}(\mathsf{x})\}
      }\,(\text{seq})
    }{
      \{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{v} * \operatorname{list}_{\beta}(\mathsf{v})\}\ \texttt{y:=*x;}\ \ldots\ \{\operatorname{list}_{\alpha}(\mathsf{x})\}
    }\,(\text{load})
  }{
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  }\,(\text{case } \textbf{list})
  \qquad
  \dfrac{}{
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}{
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        \qquad
        \{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{shuf(y);}\ \{\mathsf{list}_\alpha(\mathsf{x})\}
      }{\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{seq})
    }{\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{v} * \mathsf{list}_\beta(\mathsf{v})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{load})
  }{\{\mathsf{x} \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{case list})
  \qquad
  \cfrac{}{\{\mathsf{x} = \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \epsilon\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\models)
}{
  \cfrac{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{if\ x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{shuffle(x)}\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{proc})
}\ (\text{if})
$$

A Cyclic Termination Proof for **shuffle**

`proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }`

$\{\mathsf{list}_\beta(\mathsf{y})\}$ `rev(y);` $\{\mathsf{list}_\beta(\mathsf{y})\}$

——— (frame)

$\left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}$ `rev(y);` $\left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}$ $\qquad$ $\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}$ `shuf(y);` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

——— (seq)

$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}$ `rev(y); ...` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

——— (load)

$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{v} * \mathsf{list}_\beta(\mathsf{v})\}$ `y:=*x; ...` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

——— (case **list**) $\qquad$ ——— ($\models$)

$\{\mathsf{x} \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}$ `y:=*x; ...` $\{\mathsf{list}_\alpha(\mathsf{x})\}$ $\qquad$ $\{\mathsf{x} = \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \epsilon\ \{\mathsf{list}_\alpha(\mathsf{x})\}$

——— (if)

$\{\mathsf{list}_\alpha(\mathsf{x})\}$ `if x!=nil ...` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

——— (proc)

$\{\mathsf{list}_\alpha(\mathsf{x})\}$ `shuffle(x)` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

A Cyclic Termination Proof for **shuffle**

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proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }
```

$$\{\mathsf{list}_\beta(\mathsf{y})\}\ \texttt{rev(y);}\ \{\mathsf{list}_\beta(\mathsf{y})\}$$

$$\frac{\{\mathsf{list}_\beta(\mathsf{y})\}\ \texttt{rev(y);}\ \{\mathsf{list}_\beta(\mathsf{y})\}}{\left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}\ \texttt{rev(y);}\ \left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}}\ \text{(frame)}$$

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$$\frac{\cdots \quad \cdots}{\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \texttt{rev(y);}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ \text{(seq)}$$

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$$\frac{\cdots \quad \cdots}{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \texttt{if x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ \text{(if)}$$

$$\frac{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \texttt{if x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \texttt{shuffle(x)}\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ \text{(proc)}$$

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proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }
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$$\{\mathsf{list}_\beta(y)\}\ \texttt{rev(y);}\ \{\mathsf{list}_\beta(y)\}$$

(frame)

$$\left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\}\ \texttt{rev(y);}\ \left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\}$$

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(seq)

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(load)

$$\{\beta < \alpha \wedge x \mapsto v * \mathsf{list}_\beta(v)\}\ \texttt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(x)\}$$

(case list)

$$\{x \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(x)\}\ \texttt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(x)\}$$

($\models$)

$$\{x = \mathsf{nil} \wedge \mathsf{list}_\alpha(x)\}\ \epsilon\ \{\mathsf{list}_\alpha(x)\}$$

(if)

$$\{\mathsf{list}_\alpha(x)\}\ \texttt{if x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(x)\}$$

(proc)

$$\{\mathsf{list}_\alpha(x)\}\ \texttt{shuffle(x)}\ \{\mathsf{list}_\alpha(x)\}$$

```
proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }
```

$$\frac{\{\mathsf{list}_\alpha(x)\}\ \mathtt{shuf}(x);\ \{\mathsf{list}_\alpha(x)\}}{\{\mathsf{list}_\beta(y)\}\ \mathtt{shuf}(y);\ \{\mathsf{list}_\beta(y)\}}\ (\text{subst})$$

$$\frac{\{\mathsf{list}_\beta(y)\}\ \mathtt{shuf}(y);\ \{\mathsf{list}_\beta(y)\}}{\left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\}\ \mathtt{shuf}(y);\ \left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\}}\ (\text{frame})$$

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$$\frac{\left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\}\ \mathtt{rev}(y);\ \left\{\begin{array}{c}\beta < \alpha \wedge x \mapsto y \\ * \ \mathsf{list}_\beta(y)\end{array}\right\} \qquad \{\beta < \alpha \wedge x \mapsto y * \mathsf{list}_\beta(y)\}\ \mathtt{shuf}(y);\ \{\mathsf{list}_\alpha(x)\}}{\{\beta < \alpha \wedge x \mapsto y * \mathsf{list}_\beta(y)\}\ \mathtt{rev}(y);\ \ldots\ \{\mathsf{list}_\alpha(x)\}}\ (\text{seq})$$

$$\frac{\{\beta < \alpha \wedge x \mapsto y * \mathsf{list}_\beta(y)\}\ \mathtt{rev}(y);\ \ldots\ \{\mathsf{list}_\alpha(x)\}}{\{\beta < \alpha \wedge x \mapsto v * \mathsf{list}_\beta(v)\}\ \mathtt{y:=*x};\ \ldots\ \{\mathsf{list}_\alpha(x)\}}\ (\text{load})$$

$$\frac{\{\beta < \alpha \wedge x \mapsto v * \mathsf{list}_\beta(v)\}\ \mathtt{y:=*x};\ \ldots\ \{\mathsf{list}_\alpha(x)\}}{\{x \neq nil \wedge \mathsf{list}_\alpha(x)\}\ \mathtt{y:=*x};\ \ldots\ \{\mathsf{list}_\alpha(x)\}}\ (\text{case } \mathsf{list}) \qquad \frac{}{\{x = nil \wedge \mathsf{list}_\alpha(x)\}\ \epsilon\ \{\mathsf{list}_\alpha(x)\}}\ (\models)$$

$$\frac{\{x \neq nil \wedge \mathsf{list}_\alpha(x)\}\ \mathtt{y:=*x};\ \ldots\ \{\mathsf{list}_\alpha(x)\} \qquad \{x = nil \wedge \mathsf{list}_\alpha(x)\}\ \epsilon\ \{\mathsf{list}_\alpha(x)\}}{\{\mathsf{list}_\alpha(x)\}\ \mathtt{if\ x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(x)\}}\ (\text{if})$$

$$\frac{\{\mathsf{list}_\alpha(x)\}\ \mathtt{if\ x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(x)\}}{\{\mathsf{list}_\alpha(x)\}\ \mathtt{shuffle}(x)\ \{\mathsf{list}_\alpha(x)\}}\ (\text{proc})$$

`proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }`

$$\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{shuf(x);}\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

(subst)

$$\{\mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{shuf(y);}\ \{\mathsf{list}_\beta(\mathsf{y})\}$$

(frame)

$$\left\{\begin{array}{c}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{array}\right\}\ \mathtt{shuf(y);}\ \left\{\begin{array}{c}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \ \mathsf{list}_\beta(\mathsf{y})\end{array}\right\}$$

(conseq)

$$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{shuf(y);}\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

$$\{\mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \{\mathsf{list}_\beta(\mathsf{y})\}$$

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(seq)

$$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

(load)

$$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{v} * \mathsf{list}_\beta(\mathsf{v})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

(case **list**)

$$\{\mathsf{x} \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

($\models$)

$$\{\mathsf{x} = \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \epsilon\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

(if)

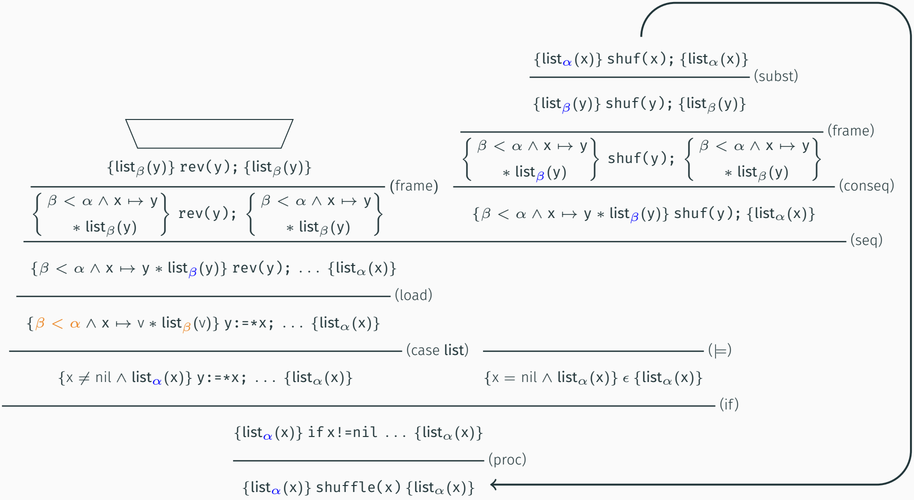
$$\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{if\ x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

(proc)

$$\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{shuffle(x)}\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

A Cyclic Termination Proof for **shuffle**

```
proc shuffle(x) { if x!=nil { y:=*x; reverse(y); shuffle(y); } }
```



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  - Requires deciding entailment of sets of constraints $\alpha < \beta$

Empirical Evaluation: Comparison with HipTNT+

Benchmark	Time (seconds)	
	HipTNT+	Cyclist
traverse acyclic linked list	0.31	0.02
traverse cyclic linked list	0.52	0.02
append acyclic linked lists	0.36	0.03
TPDB Shuffle	1.79	0.21
TPDB Alternate	6.33	1.47
TPDB UnionFind	4.03	1.21

Empirical Evaluation: Comparison with AProVE

Benchmark Suite	Test	Time (seconds) AProVE	Time (seconds) CYCLIST
Costa_Julia_09-Recursive	Ackermann	3.82	0.14
	BinarySearchTree	1.41	0.95
	BTree	1.77	0.03
	List	1.43	1.74
Julia_10-Recursive	AckR	3.22	0.14
	BTreeR	2.68	0.03
	Test8	2.95	0.97
AProVE_11_Recursive	CyclicAnalysisRec	2.61	5.21
	RotateTree	5.86	0.32
	SharingAnalysisRec	2.47	4.72
	UnionFind	TIMEOUT	1.21
BOG_RTA_11	Alternate	5.47	1.47
	AppE	2.19	0.09
	BinTreeChanger	3.38	3.33
	CAppE	2.04	1.78
	ConvertRec	3.72	0.06
	DupTreeRec	4.18	0.03
	GrowTreeR	3.53	0.05
	MirrorBinTreeRec	4.96	0.02
	MirrorMultiTreeRec	5.16	0.63
	SearchTreeR	2.74	0.34
	Shuffle	11.72	0.21
	TwoWay	1.94	0.02

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  - Constraints on explicit approximations
  - Entire pre-/post-conditions (bi-abduction)

github.com/ngorogiannis/cyclist