Automatic Cyclic Termination Proofs for Recursive Procedures in Separation Logic

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Automatically Proving Termination: Challenges

```
proc shuffle(x) {
    if x != nil {
        y := *x;
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heap manipulation

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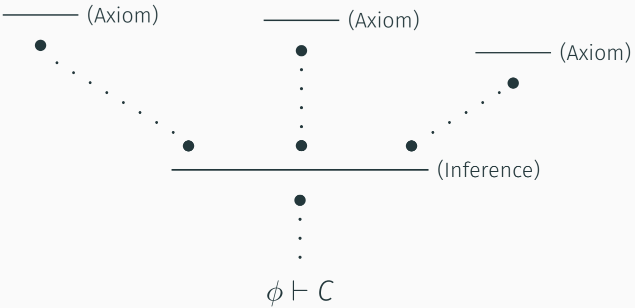
- Following the approach of Brotherston et al. (POPL ’08)



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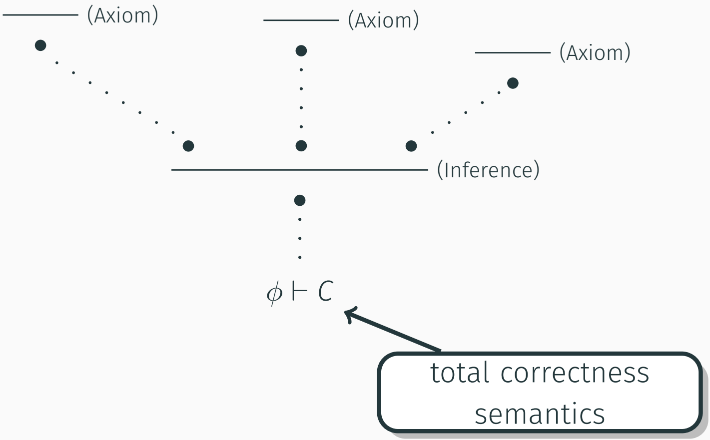

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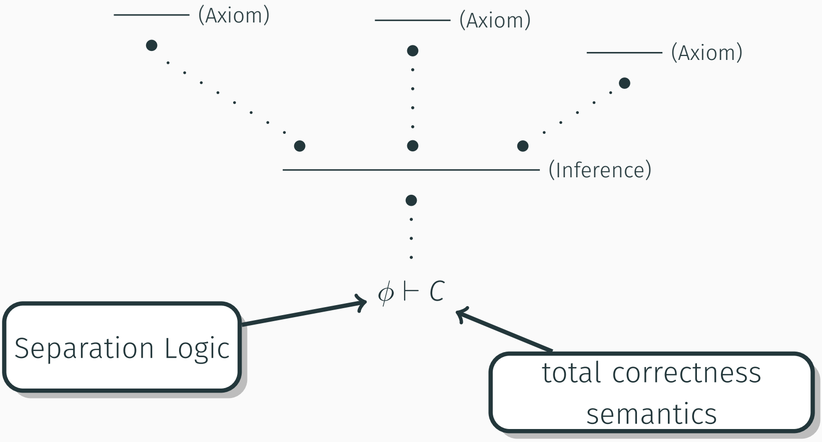

Automatically Proving Termination using Cyclic Proof

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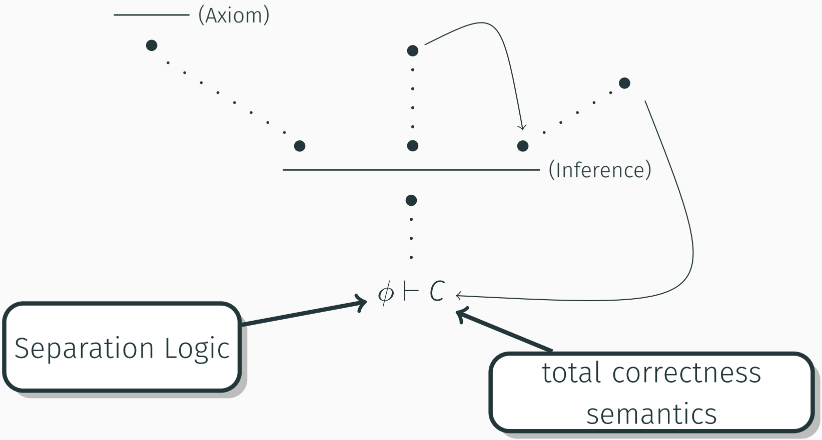

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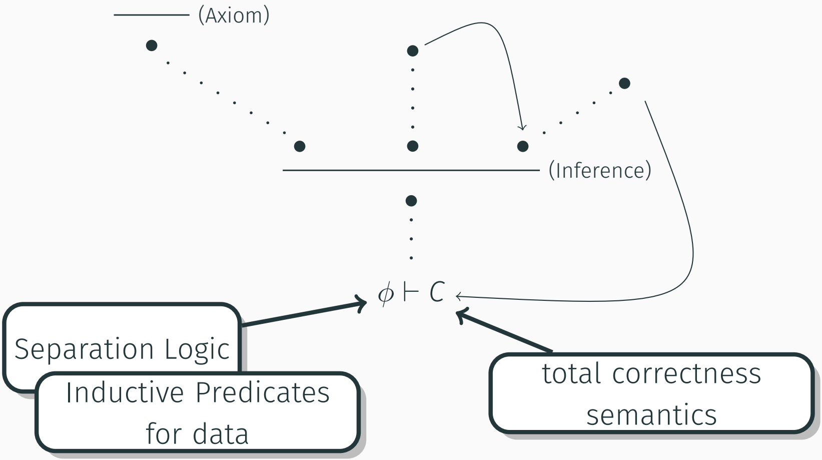

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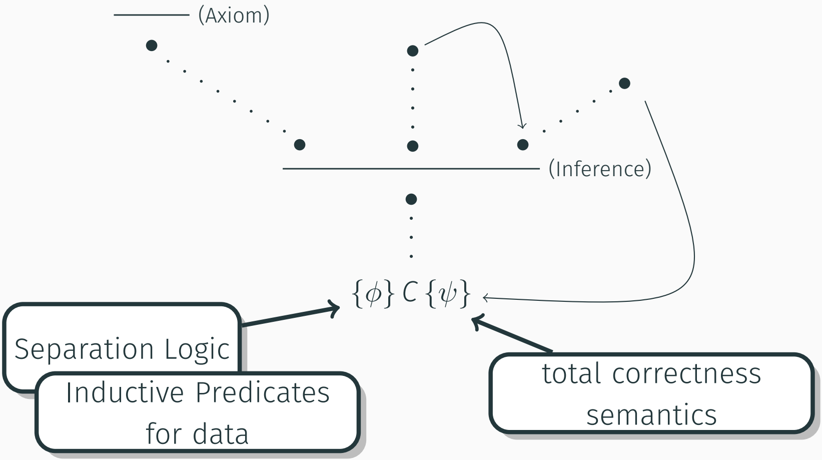

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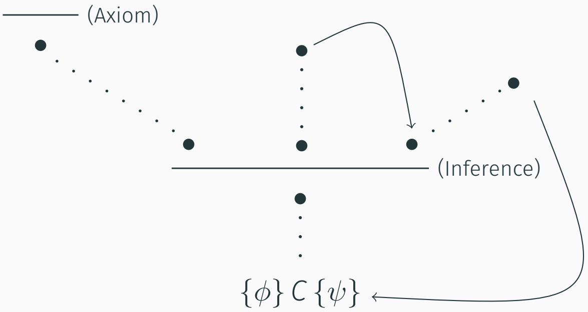

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- Termination measures extracted *automatically*

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- *Predicates* $P(x_1, \ldots, x_n)$ describe specific structures

  e.g. $\mathbf{lseg}(x, y)$, $\mathbf{list}(z)$
- *Symbolic heap* syntax makes reasoning easier

$$x = y \land z \neq \text{nil} \land \mathbf{lseg}(x, y) * \mathbf{list}(z) * v \mapsto w$$

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- We support user-defined inductive predicates, e.g.

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- We achieve automation by implementing the proof system in CYCLIST
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  - Requires deciding entailment of sets of constraints $\alpha < \beta$
- Currently, we need to provide procedure summaries

A Cyclic Termination Proof for `shuffle`

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$$\dfrac{\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{\beta < \alpha \wedge \mathsf{x} \mapsto v * \mathsf{list}_\beta(v)\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{load})$$

$$\dfrac{\{\beta < \alpha \wedge \mathsf{x} \mapsto v * \mathsf{list}_\beta(v)\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{x \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{case } \mathbf{list})$$

$$\dfrac{}{\{x = \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \epsilon\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\models)$$

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$$\dfrac{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{if\ x!=nil}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{shuffle(x)}\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\text{proc})$$

A Cyclic Termination Proof for `shuffle`

`proc shuffle(x)` $\{\mathsf{list}_\alpha(\mathsf{x})\}$ `{`

`if x!=nil { y:=*x; reverse(y); shuffle(y); } }` $\{\mathsf{list}_\alpha(\mathsf{x})\}$

$$\frac{\{\mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{shuf(x);}\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{\mathsf{list}\ (\mathsf{y})\}\ \mathtt{shuf(y);}\ \{\mathsf{list}\ (\mathsf{y})\}}\ \text{(subst)}$$

$$\frac{\{\mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \{\mathsf{list}_\beta(\mathsf{y})\}}{\left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}\ \mathtt{rev(y);}\ \left\{\begin{matrix}\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} \\ * \mathsf{list}_\beta(\mathsf{y})\end{matrix}\right\}}\ \text{(frame)}$$

$$\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{shuf(y);}\ \{\mathsf{list}_\alpha(\mathsf{x})\}$$

$$\frac{\cdots \qquad \cdots}{\{\beta < \alpha \wedge \mathsf{x} \mapsto \mathsf{y} * \mathsf{list}_\beta(\mathsf{y})\}\ \mathtt{rev(y);}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ \text{(seq)}$$

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$$\frac{\{\beta < \alpha \wedge \mathsf{x} \mapsto v * \mathsf{list}_\beta(v)\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}{\{x \neq \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \mathtt{y:=*x;}\ \ldots\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ \text{(case list)}$$

$$\frac{}{\{x = \mathsf{nil} \wedge \mathsf{list}_\alpha(\mathsf{x})\}\ \epsilon\ \{\mathsf{list}_\alpha(\mathsf{x})\}}\ (\models)$$

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A Cyclic Termination Proof for shuffle

proc shuffle(x) $\{\mathsf{list}_\alpha(x)\}$ {
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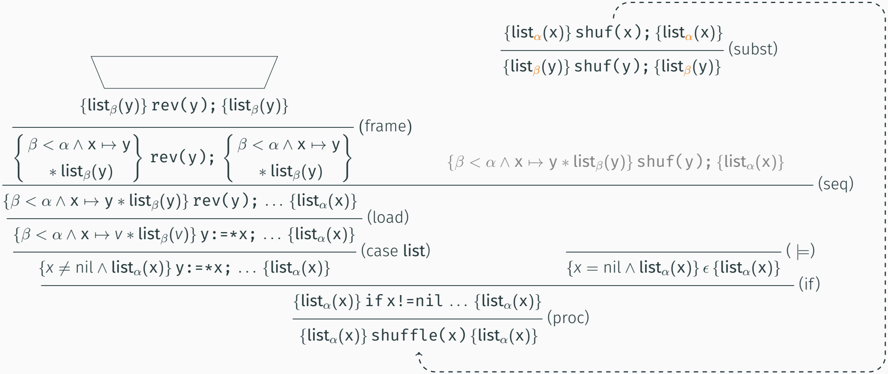

A Cyclic Termination Proof for `shuffle`

$$\texttt{proc shuffle(x)}\ \{\mathsf{list}_\alpha(x)\}\ \{$$
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$$\dfrac{\{\mathsf{list}_\alpha(x)\}\ \texttt{shuf(x);}\ \{\mathsf{list}_\alpha(x)\}}{\{\mathsf{list}_\beta(y)\}\ \texttt{shuf(y);}\ \{\mathsf{list}_\beta(y)\}}\ (\text{subst})$$

$$\dfrac{\{\mathsf{list}_\beta(y)\}\ \texttt{rev(y);}\ \{\mathsf{list}_\beta(y)\}}{\left\{\begin{matrix}\beta < \alpha \wedge x \mapsto y\\ * \mathsf{list}_\beta(y)\end{matrix}\right\}\ \texttt{rev(y);}\ \left\{\begin{matrix}\beta < \alpha \wedge x \mapsto y\\ * \mathsf{list}_\beta(y)\end{matrix}\right\}}\ (\text{frame})$$

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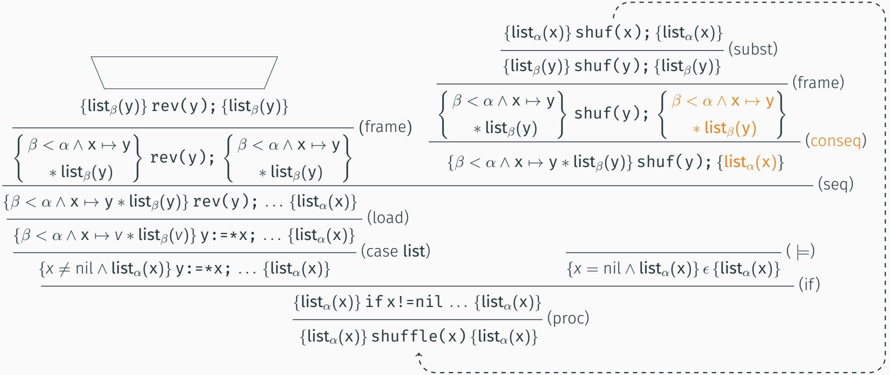
A Cyclic Termination Proof for shuffle

```
proc shuffle(x) {list_α(x)} {
     if x!=nil { y:=*x; reverse(y); shuffle(y); } } {list_α(x)}
```



A Cyclic Termination Proof for `shuffle`

proc shuffle(x) $\{\mathsf{list}_\alpha(\mathsf{x})\}$ {
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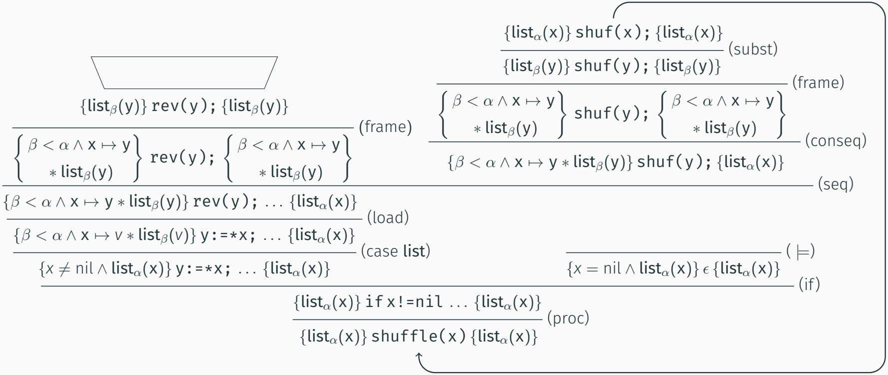

A Cyclic Termination Proof for `shuffle`

$\texttt{proc shuffle(x)}\ \{\mathsf{list}_\alpha(\mathsf{x})\}\ \{$

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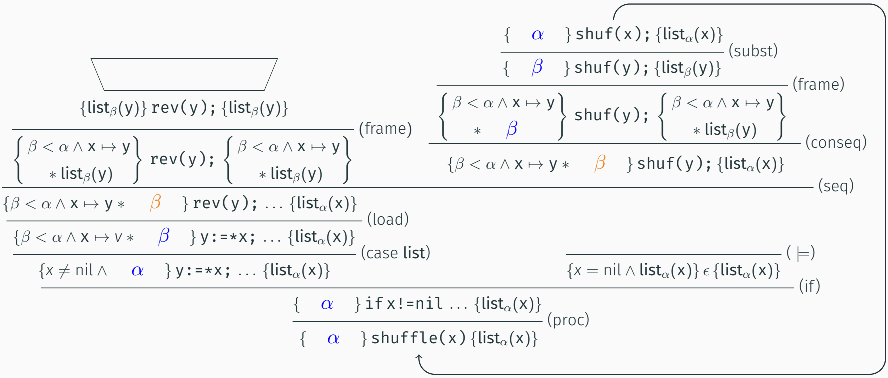

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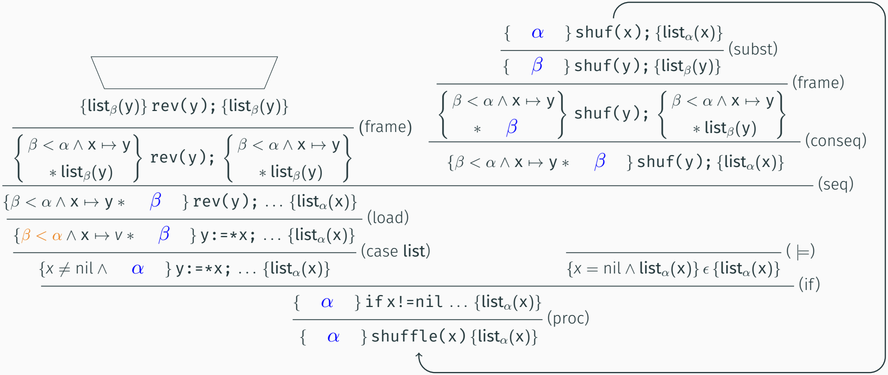

Some Related Tools

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Empirical Evaluation: Comparison with HipTNT+

Benchmark test	Time (sec) / % Annotated	
	HipTNT+	Cyclist
traverse acyclic linked list	0.31 (25%)	0.02 (33%)
traverse cyclic linked list	0.52 (29%)	0.02 (38%)
append acyclic linked lists	0.36 (25%)	0.03 (10%)
TPDB Shuffle	1.79 (22%)	0.21 (29%)
TPDB Alternate	6.33 (13%)	1.47 (12%)
TPDB UnionFind	4.03 (26%)	1.21 (25%)

Empirical Evaluation: Comparison with AProVE

		Time (seconds)		
Benchmark Suite	Test	AProVE	CYCLIST	(% Annot.)
Costa_Julia_09-Recursive	Ackermann	3.82	0.14	(18%)
	BinarySearchTree	1.41	0.95	(13%)
	BTree	1.77	0.03	(22%)
	List	1.43	1.74	(19%)
Julia_10-Recursive	AckR	3.22	0.14	(18%)
	BTreeR	2.68	0.03	(22%)
	Test8	2.95	0.97	(13%)
AProVE_11_Recursive	CyclicAnalysisRec	2.61	5.21	(27%)
	RotateTree	5.86	0.32	(14%)
	SharingAnalysisRec	2.47	4.72	(16%)
	UnionFind	TIMEOUT	1.21	(25%)
BOG_RTA_11	Alternate	5.47	1.47	(12%)
	AppE	2.19	0.09	(23%)
	BinTreeChanger	3.38	3.33	(20%)
	CAppE	2.04	1.78	(25%)
	ConvertRec	3.72	0.06	(38%)
	DupTreeRec	4.18	0.03	(20%)
	GrowTreeR	3.53	0.05	(20%)
	MirrorBinTreeRec	4.96	0.02	(22%)
	MirrorMultiTreeRec	5.16	0.63	(33%)
	SearchTreeR	2.74	0.34	(14%)
	Shuffle	11.72	0.21	(29%)
	TwoWay	1.94	0.02	(25%)

Future Work

- More expressive contraints for predicate approximations

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    - Predicate label annotations

Future Work

- More expressive contraints for predicate approximations
- Can we infer procedure specifications?
    - Predicate label annotations
    - Entire pre-/post-conditions (bi-abduction)

Thank You

`github.com/ngorogiannis/cyclist`