Positive and Negative Preferences

Francesca Rossi
University of Padova, Italy

Joint work with:
S. Bistarelli,
University "G. d'Annunzio", Pescara, and
Istituto di Informatica e Telematica, CNR, Pisa, Italy
M.S. Pini, K.B. Venable
University of Padova, Italy

Outline

- Motivation: I like this, I don’t like that!
- Background: semiring-based soft constraints
- Negative Preferences
- Positive Preferences
- Bipolar Preferences
- Bipolar preference problems
- Solving bipolar preference problems

Motivation and desires

- In many real life scenarios users express degrees of acceptance and rejection
- Positive and negative preferences, although symmetric, need different combination operators
  - Example: I like red cars, I like convertibles: red convertible is the best choice
  - Example: I hate black cars, I hate SUVs: black SUV is the worse choice
- Indifference: neither positive nor negative
  - Example: I am indifferent to white cars
- Compensation:
  - a black convertible is acceptable

Soft constraints

- One way to model one kind of preferences
  - Quantitative, non-conditional
- Based on an algebraic structure
- Other kinds of preferences:
  - Qualitative, positive/negative, conditional, etc.
- Other ways to model preferences:
  - CP-nets, partial CSPs, hierarchical CSP/CLP, valued CSPs, etc.

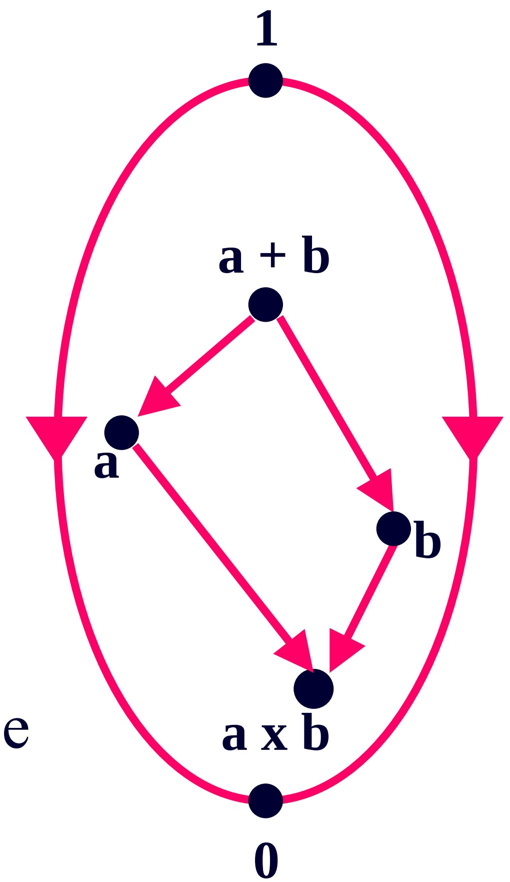
C-semiring framework for constraints

- Variables $\{X_1,\ldots,X_n\}=X$
- Domains $\{D(X_1),\ldots,D(X_n)\}=D$
- C-semiring $\langle A, +, \times, 0, 1\rangle$:
  - A set of preferences
  - $+$ additive operator commutative, associative, idempotent, unit element 0, flattening property;
  - $\times$ multiplicative operator: commutative, associative, unit element 1, absorbing element 0
  - 0,1 respect. bottom and top element
  - $\times$ distributes over $+$

Bistarelli, Montanari, Rossi 1995
(valued CSPs equivalent if total order)

Induced Ordering

- Given a c-semiring $\langle A, +, \times, 0, 1\rangle$:
- $+$ induces a partial order $\leq$ on A:
  - $a \leq b$ iff $a+b = b$
- $a+b \geq a,b$ (=lub(a,b))
  - Adding two pref.s gives a higher preference
- $a \times b \leq a,b$ (=glb(a,b) if $\times$ idempotent)
  - Combining two pref.s gives a lower preference
- $+$ and $\times$ monotone on $\leq$



Soft and Hard constraints

- Soft constraint: pair c=<f,con> where:
  - Scope: con=$\{X_{c1},\ldots, X_{ck}\}$ subset of X
  - Preference function :

    f: $D(X_{c1})x\ldots xD(X_{ck}) \rightarrow A$

    tuple $(v_1,\ldots, v_k) \rightarrow p$ preference

- Hard constraint: soft constraint where for each tuple (v1,…, vk)

  $f(v_1,\ldots, v_k)=0$ the tuple is forbidden

  or $f(v_1,\ldots, v_k)=1$ the tuple is allowed

Soft Constraint Satisfaction Problems

Set of variables V
Set of **hard and soft constraints**

Solution: Assignment to all the variables in V

Preference of solution S:

$pref(S)=f_1(S_1) \times \ldots \times f_n(S_n)$

f_i = preference function of i-th constraint

S_i = projection of S on i-th constraint

Ordering: the preference relation **on the set of solutions** is a partial order

Optimal solution S': there is no S, pref(S)>pref (S')

Fuzzy-SCSP example

Fuzzy semiring

$S = \langle A, +, \times, \mathbf{0}, \mathbf{1} \rangle$

$S_{FCSP} = \langle [0,1], max, min, 0, 1 \rangle$

{Fish, Meat} {White, red}

(Fish, white) → 1 (Meat, white) → 0.3

(Fish, red) → 0.8 (Meat, red) → 0.7

{12 am, 1 pm} {2 pm, 3 pm}

(12 am, 2 pm) → 1 (1 pm, 2 pm) → 0

(12 am, 3 pm) → 1 (1 pm , 3 pm) → 1

Solution *S*

Lunch = 1 pm
Main course = meat
Wine = white
Swim = 2 pm

pref(S) = min(0.3,0) = 0

Solution *S'*

Lunch = 2 pm
Main c
W
Sw = 2 pm
white

OPTIMAL

pref(S) = min(1,1) = 1

Negative Preferences

- Standard c-semiring $<A, +, \times, 0, 1>$
- $a \times b \leq a,b$: combining negative preferences should go down in the ordering

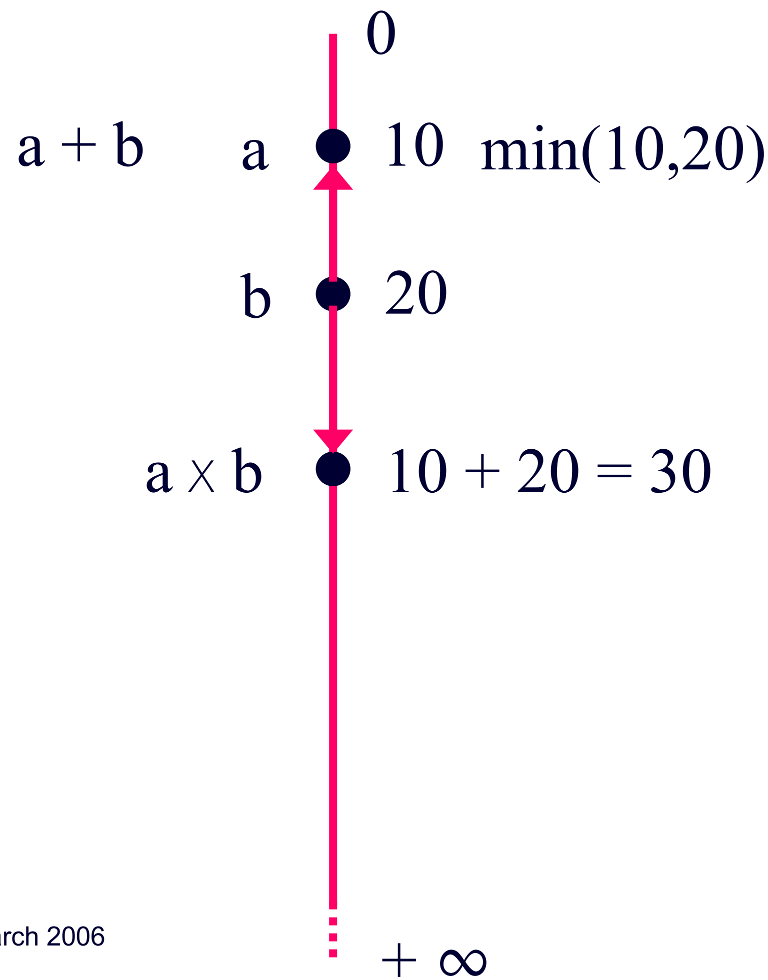
  Example:
  - Weighted semiring $<\mathbb{R}^+, \min, +, +\infty, 0>$,
    - Values = costs
    - Goal = minimize the sum of costs

Weighted c-semiring: induced ordering

$$\langle A, \; + , \; \times , \; 0 , \; 1 \rangle$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\langle \mathbb{R}^{+}, \min , + , +\infty , 0 \rangle$$

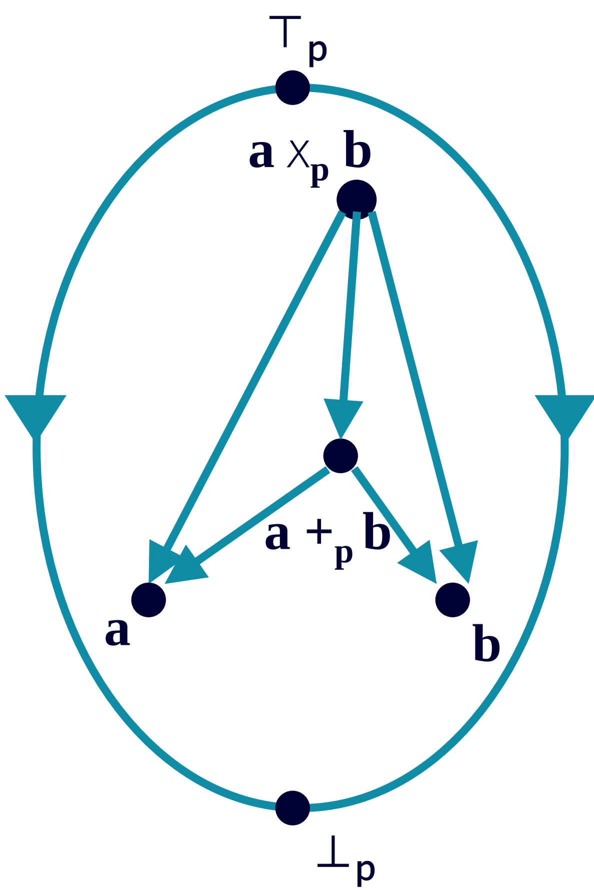


Positive Preferences

- C-semiring not adapt because the combining positive preference should go up in the ordering → new structure
- Positive Preference Structure $<P, +_p, \times_p, \bot_p, \top_p>$
- P set, $\bot_p, \top_p \in P$;
  - $+_p$ additive operator: commutative, associative, idempotent, unit element $\bot_p$, absorbing element $\top_p$;
  - $\times_p$ multiplicative operator: commutative, associative, unit element $\bot_p$, absorbing element $\top_p$
  - $\bot_p$, $\top_p$ respect. bottom and top element
  - $\times_p$ distributes over $+_p$

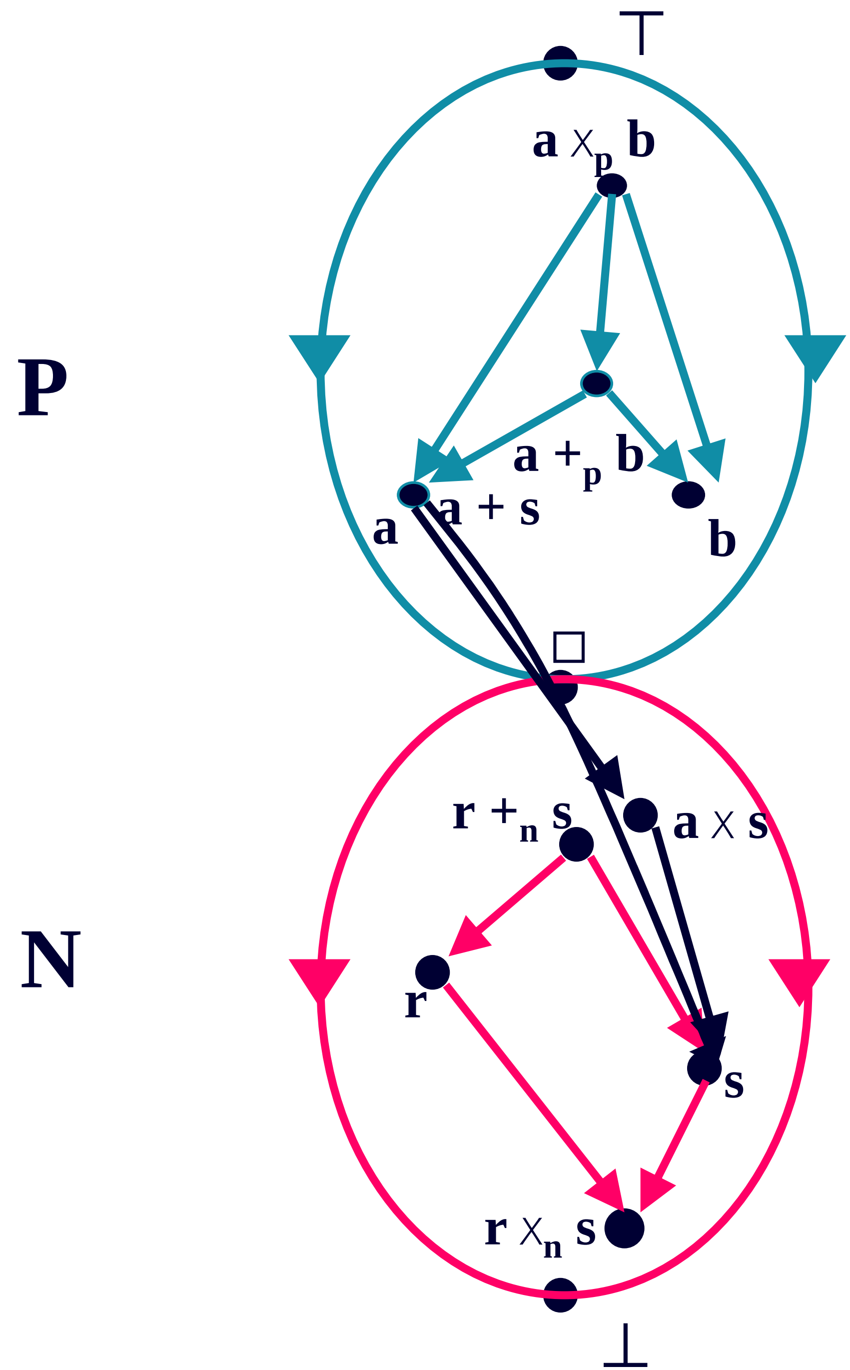
Positive Preference Structure: Induced Ordering

- Given Positive Preference Structure $<P, +_p, \times_p, \bot_p, \top_p>$
- $+p$ induces a partial order $\leq$ on P:
  - $a \leq b$ iff $a +_p b = b$
- $a +_p b \geq a,b$ (=lub(a,b))
  - Adding two pref.s gives a higher preference
- $a \times_p b \leq a,b$ (=glb(a,b) if $\times_p$ idempotent)
  - Combining two pref.s gives a HIGHER preference
- $+_p$ and $\times_p$ monotone on $\leq$



Bipolar preferences

- Preference Structure
  $<P \cup N, +_p, \times_p, +_n, \times_n, +, \times, \bot, \square, \top>$
  - $<P, +_p, \times_p, \square, \top>$ is a positive preference structure
  - $<N, +_n, \times_n, \bot, \square>$ is a c-semiring
  - $+: (P \cup N)^2 \rightarrow (P \cup N)$
    - $+|_N = +_n$,
    - $+|_P = +_p$,
    - $a+s=a$, for all $a \in P$ and $s \in N$
  - $\times: (P \cup N)^2 \rightarrow (P \cup N)$,
    - $\times|_N = \times_n$,
    - $\times|_P = \times_p$,
    - $s \leq a \times s \leq a$, for all $a \in P$ and $s \in N$ (**compensation**, plus other properties defined later)



Preference Structure

$<P \cup N, +_p, \times_p, +_n, \times_n, +, \times, \bot, \Box, \top>$

$<P, +_p, \times_p, \Box, \top>$ positive pref. struct.

$<N, +_n, \times_n, \bot, \Box>$ is a c-semiring

$+$ induces a partial order

$\forall\ v \in P \cup N \quad \bot \leq v \leq \top$

$\forall\ p \in P,\ \forall n \in N,\ n \leq \Box \leq p$

$\forall\ a, b \in P \quad a + b = a +_p b \geq a,b$

$\forall\ r, s \in N \quad r + s = r +_n s \geq r,s$

$\forall\ a, b \in P \quad a \times b = a \times_p b \geq a,b$

$\forall\ r, s \in N \quad r \times s = r \times_n s \leq r,s$

$\forall\ a \in P,\ \forall s \in N,\ a + s = a$

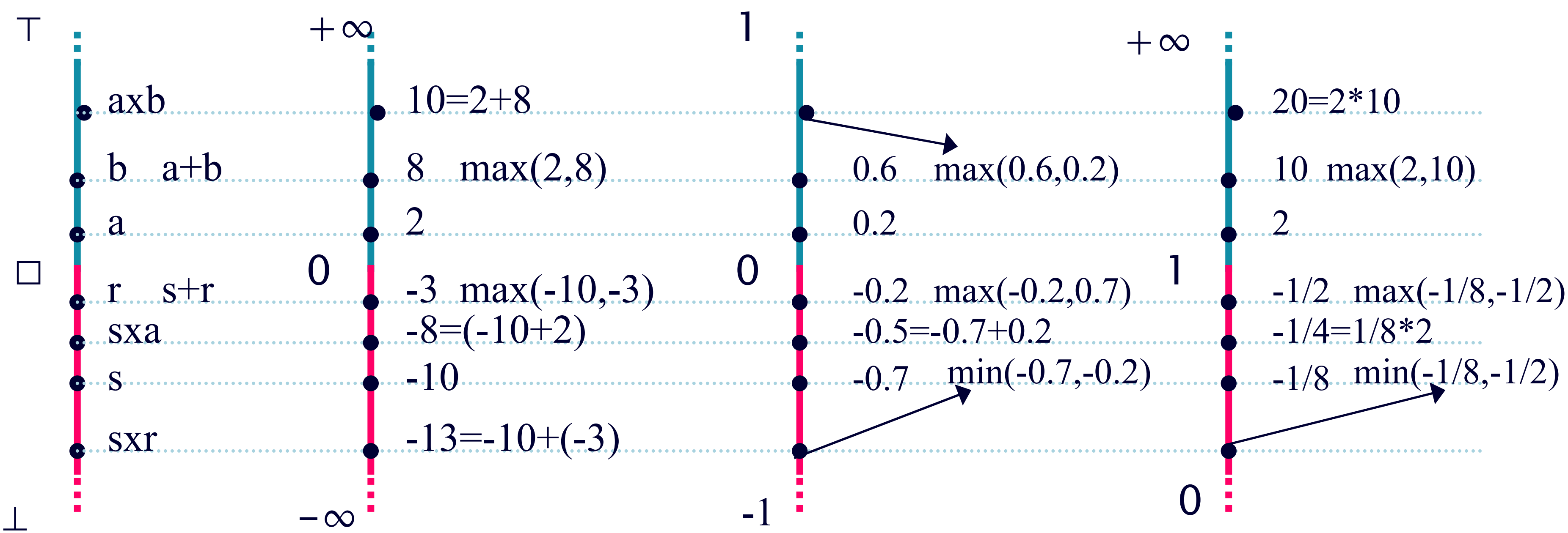
$\forall\ a \in P,\ \forall s \in N,\ s \leq a \times s \leq a$

Examples of unipolar structures

- Negative structures: any c-semiring
  - Fuzzy <[0,1],max,min,0,1>
  - Weighted <R+,min,+,+∞,0>
  - Probabilistic <[0,1],max, x, 0,1>
- Positive structures
  - <R+, max, +, 0, +∞>
  - <[0,1],max,max,0,1>
  - <[1,+∞],max, x, 0, +∞>

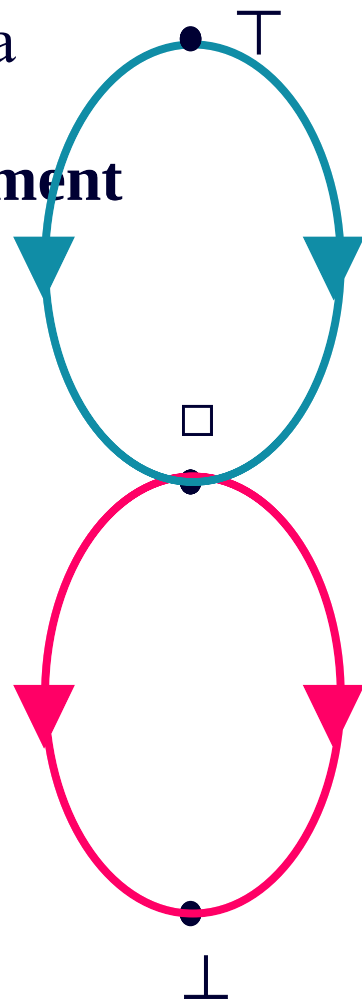
Examples of bipolar preference structures

P∪N	$+_p$	$\times_p$	$+_n$	$\times_n$	$+$	$\times$	$\bot$	$\Box$	$\top$
$[0,+\infty]$ $\cup[0,-\infty]$	max	+	max	+	max	+	$-\infty$	0	$+\infty$
$[0,1]$ $\cup[-1,0]$	max	max	max	min	max	+	-1	0	1
$\mathbb{Z}^{>1}$ $\cup[0,1]$	max	*	max	min	max	*	0	1	$+\infty$



Conservative extension of soft constraints

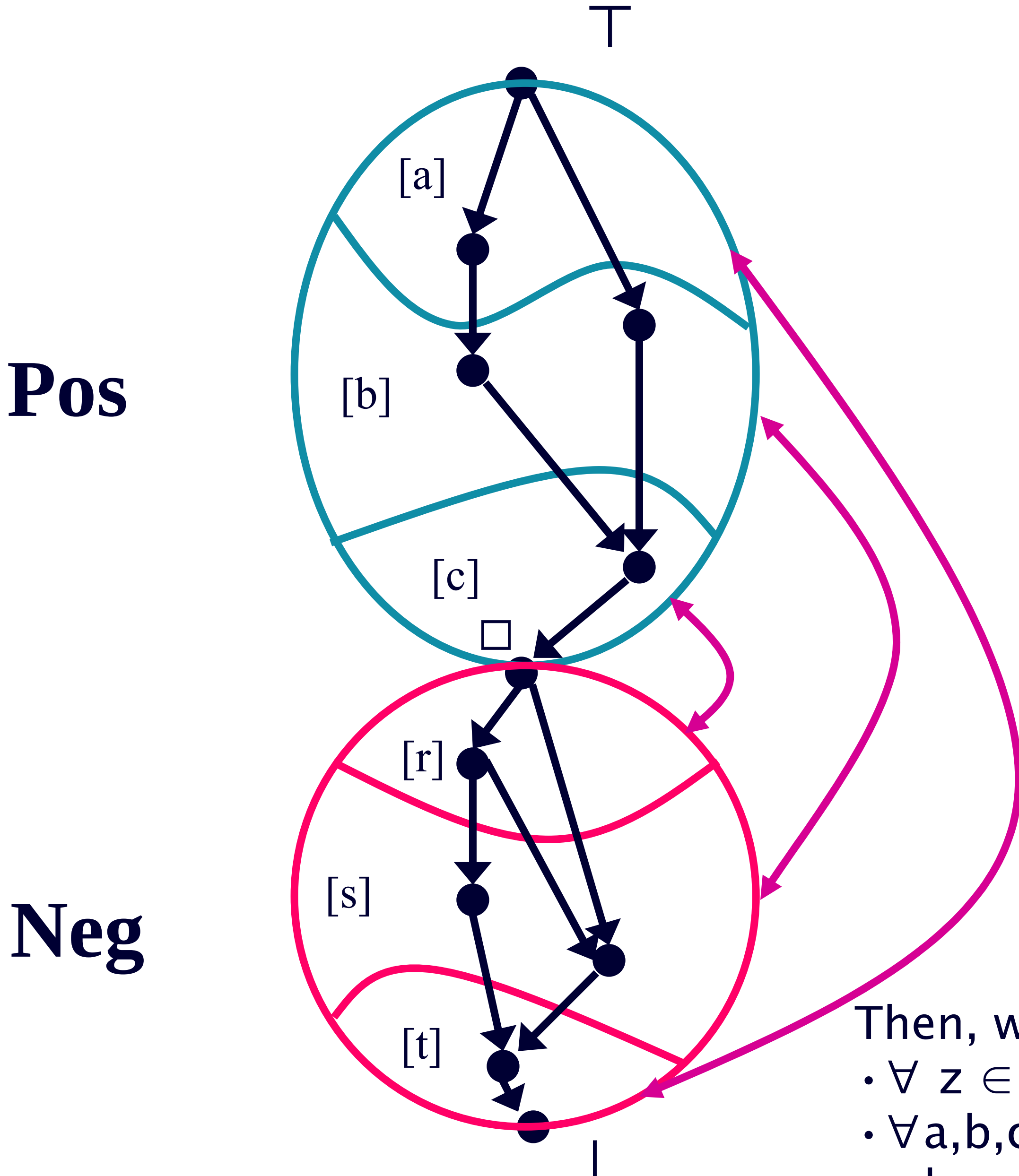
- The preference structure represented by a c-semiring is a special case of a bipolar preference structure where: **the positive preferences collapses into the indifferent element**
- Examples:
- Fuzzy semiring <[0,1],max,min,0,1>
  - preference = 1 → indifference, nothing wrong
  - preference = 0.9 → weak rejection
  - preference = 0.2 → strong rejection
  - preference = 0 → unacceptable
- Classic semiring <{false,true}, or, and, false, true>
  - preference = true → indifference, no violations
  - preference = false → rejection, some violations



Other properties for preference compensation

- To drfine them:

1. Partition of P and N in the same number of classes
2. Elements in the same class behave similarly when combined with elements of the opposite class
3. Ordering among the classes
4. Correspondence function $c \rightarrow c^{-1}$, for all h in c and for all f in c^{-1}, $h \times f = \square$



1. Partition of P and N in the same number of classes

2. Order the classes
   $[a] > [b]$ iff $\forall x \in [a], \forall y \in [b], x \geq y$

$[a] \bowtie [b] > [c] > [r] > [s] > [t]$

3. Define a bijection f such that
   $\forall x,y \in N$,
   $[x] \leq$ or $\bowtie$ $[y]$ iff $f([x]) \geq$ or $\bowtie f([y])$
   $\bowtie$ incomparable

Then, we have:

- $\forall z \in [x], \forall y \in f([x]), z \times y = \square$
- $\forall a,b,c \in P \cup N$, if $[a] < [b]$ then $a \times c < b \times c$ and $a < a \times b < b$

Associativity of $\times$

- $\times$ might be not associative

- Example: If $\times_p$ idempotent,

  $p \in P$, $n \in N$ and $p \times n = \square$.

  Then

  $p \times (p \times n) = p \times_p \square = p$

  $(p \times_p p) \times n = p \times n = \square$

- It may be not associative even if $\times_p$ and $\times_n$ are not idempotent

Associativity of $\times$

$P \cup N$	$+_p$	$\times_p$	$+_n$	$\times_n$	$+$	$\times$	$\bot$	$\square$	$\top$
$[0,+\infty] \cup [0,-\infty]$	max	+	max	+	max	+	$-\infty$	0	$+\infty$
$[0,1] \cup [-1,0]$	max	max	max	min	max	+	-1	0	1
$\mathbb{Z}^{>1} \cup [0,1]$	max	*	max	min	max	*	0	1	$+\infty$

- The first structure is associative, while the other two are not

Associativity of x

- If one of the following holds:
  - x_p or x_n idempotent
  - $|P| \neq |N|$
  - there are negative (positive) preferences a and b such that $a,b \neq \perp$ ($\top$) and $a \; x_n \; b = \perp$ ($\top$)
  - there are preferences a,b,c such that

    $a \; x \; f(b) = c$ but $c \; x \; b \neq a$
- x is not associative

Bipolar Preference Problems (1)

- Variables $\{X_1,\ldots,X_n\}=X$
- Domains $\{D(X_1),\ldots,D(X_n)\}=D$
- Preference Structure $<P \cup N, +_p, \times_p, \;, +_n, \times_n, +, \times, \bot, \Box, \top>$
- Set of bipolar constraints C
  - Bipolar constraint: pair $c=<f,con>$ where:
    - Scope: $con=\{X_{c1},\ldots, X_{ck}\}$ subset of X
    - Preference function :

      $f: \quad D(X_{c1}) x \ldots x D(X_{ck}) \rightarrow P \cup N$

      tuple $(v_1,\ldots,v_k) \rightarrow d$ degree of preference or rejection

Bipolar Preference Problems (2)

Solution: Assignment to all the variables in V

Preference of solution S:

pref(S)=$(p_1 \times_p \ldots \times_p p_k)$ x $(n_1 \times_n \ldots \times_n n_h)$

$p_i \in P$ positive values assigned to projections

$n_i \in N$ negative values assigned to projections

Ordering: the preference relation on the set of solutions is a partial order

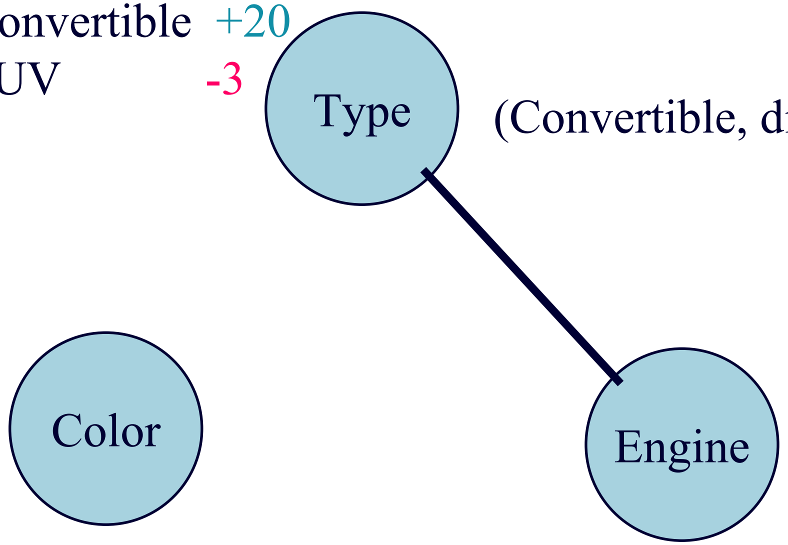
Optimal solution S': there is no S, pref(S)>pref (S')

Example

P∪N	$+_p$	$\times_p$	$+_n$	$\times_n$	$+$	$\times$	$\bot$	$\square$	$\top$
$[0,+\infty] \cup [0,-\infty]$	max	+	max	+	max	+	$-\infty$	0	$+\infty$

I like red cars,
I like convertibles a lot;
I hate black cars,
I don't care much for SUVs,
I like diesel because it is cheaper
But I don't want a diesel convertible
I am indifferent to white cars

Convertible +20
SUV -3



(Convertible, diesel) -20

Red +10
Black -10
White 0

Diesel +10
Gasoline 0

d=diesel


20+10= 30


20

d 
-3+10+10= 17


10

d 

7

d 
0


d 
-3

d 
-10


-13

d 
-30

Solving bipolar preference problems

- Branch and bound can be adapted
- Value associated at every tree node: (p,n)
- To compute an upper bound of the preference of solutions in the subtree:
  - Compute (p’,n’), where p’ (n’) combination of best positive (negative) preferences in subtree
  - Upper bound = (pxp’) x (nxn’)
- If x is associative, p and n can be combined, also in the computation of the upper bound
- Constraint propagation: useful on negative preferences, not convenient on positive preferences

Open issues

- Conditions for associativity of compensation
- Negative preferences which cannot be compensated
- Branch and bound and constraint propagation when x is associative
- Connection with non-monotonic reasoning
- Tractable structures for bipolar preference problems