V. Type-theoretical semantics

- Applications of type theory
  - Proof assistants
  - Dependently typed programming
  - Modelling and formal reasoning
- In this lecture: an example in linguistic semantics
  - Type-theoretical semantics with coercive subtyping

Formal semantics in linguistics

- Questions (asked about linguistic semantics)
  - A: Is semantics objective or conceptual?
  - B: Is semantics mathematical or psychological?
- Formal semantics
  - Positive answer to B
  - What about A?
- Examples of formal semantics
  - Montague semantics (Montague 1974)
    - Objective and mathematical
  - Type-theoretical semantics (Ranta 1994, Luo 2010)
    - Conceptual (?) and mathematical
  - Others (see, eg, references in Portner & Partee 2002)

Historical remarks

- Historical developments of Montague semantics
  - Richard Montague (1930 – 1971)
  - In early 1970s: Montague, Lewis, Cresswell, Parsons, …
  - Later developments: Dowty, Partee, …
- Other formal semantics
  - Discourse Representation Theory (Kemp 1981, Heim 1982)
  - Situation semantics (Barwise & Berry 1983)

Remark on anaphora analysis:

- Donkey sentences: a (difficult) form of anaphora
- Eg, "Every farmer who owns a donkey beats it."
- $\forall x.\ farmer(x)\ \&\ [\ \exists y.\ donkey(y)\ \&\ own(x,y)\ ] \Rightarrow beat(x,?y)$

- Type-theoretical semantics
  - Ranta 1994 (not firmly considered as a logical semantics)
    - $\Pi x{:}Farmer\ \Pi z : [\Sigma y{:}Donkey.\ own(x,y)].\ beat(x, \pi_1(z))$
  - Formal semantics with TTs with canonical objects
  - Solutions to the problems caused by the limitation of Montague semantics based on Church's simple type theory
  - Potential application to NL reasoning based on the current proof technology

(RHUL project on lexical semantics in type theory:
http://www.cs.rhul.ac.uk/home/zhaohui/lexsem.html)

Montague semantics

- Semantic language of Montague semantics
  - Church's simple type theory (1940)
  - IL – Montague's "Intensional Logic" (this aspect of intensionality is omitted for simplification here.)
- Syntactic categories of NLs
  - Sentences (S): "John walks."
  - Common Nouns (CN): bank, school, book, man
  - Intransitive Verbs (IV): run, walk, talk, work
  - Adjectives (Adj): pretty, tired, handsome
  - …

Semantic types in Montague semantics

Type	**Informal explanation**
Prop	Type of truth values
e	Type of all entities
e→Prop	Type of subsets of entities

Montague's semantics of categories

Category	Semantic Type
S	Prop
CN	e→Prop
IV	e→Prop
Adj (CN/CN)	(e→Prop)↦(e→Prop)

Montague semantics: examples

- Common nouns (as functional subsets of entities)
  - man : CN
  - [man] : e → Prop
- Verbs (as predicates over entities)
  - walk : IV
  - [walk] : e → Prop
  - [John walks] = [walk](j), if j = [John] : e.
- Adjectives (as functions from subsets to subsets)
  - handsome : CN/CN
  - [handsome] : (e→Prop) → (e→Prop)
  - [handsome man] = [handsome]([man]) : e → Prop

Montague semantics: problems

- New developments in lexical semantics
  - Generative lexicon (Pustegovsky 1995)
  - Copredication (Asher 2010)
- Limitation of the Montagovian setting
  - Formalisation of new lexical theories in Montagovian setting plus subtyping (Asher & Pustejovsky. 2005; Asher 2008/2010)
  - Difficulties of the above approach (Luo 2010)
  - Reason: incompatibility of the Motagovian setting with subtyping

Example in Montague semantics

- Dot-types in Generative Lexicon (Pustejovsky 1995)
  - Example: PHY•INFO
  - PHY•INFO ≤ PHY and PHY•INFO ≤ INFO
- Examples
  - Books are both physical and informational.
    - [book] : PHY•INFO→Prop
  - "heavy" ("boring") is about physical (informational) entities.
    - [heavy] : (PHY→Prop)→(PHY→Prop)
    - [boring] : (INFO→Prop)→(INFO→Prop)
- What about "a heavy book" or "a boring book"?
  - To apply [heavy] or [boring] to [book], we would need
    PHY•INFO→Prop ≤ PHY→Prop
    PHY•INFO→Prop ≤ INFO→Prop
    NOT the case! (It is just the other way around!)

- Some new developments in, eg, lexical sem
  - Reference transfers (cf, Jackendoff)
    - "The ham sandwich shouts." (in a special context)
  - Logical polysemy (cf, Pustejovsky 95)
    - "burn a boring book" ("book": both "physical" and "informational")
- Problem and solution
  - Montague Grammar is not fit for the purpose! (cf, work by Pustejovsky, Asher, …)
  - Types instead of functional subsets + coercive subtyping

- Type-theoretical semantics – a promising approach
  - Formal semantics based on modern TTs
  - A key difference: multi-sorted (v.s. single-sorted Montagovian setting) (Ranta 1994)
- May offer solutions, but …
- New problem
  - Not enough oprns on types (as compared with oprns on functional subsets of type e→t)
  - Promising solution: coercive subtyping
  - Offers natural solutions (Luo 2010)

Type-theoretical semantics

Category	Semantic Type
S	Prop
CNs (book, man, …)	types (each CN is interpreted as a type: [book]. [man], …)
IV	A→Prop (A is the "meaningful domain" of a verb)
Adj	A→Prop (A is the "meaningful domain" of an adjective)

- CNs as types (not sets!)
  - ⟦human⟧ : Type
  - ⟦man⟧ : Type
  - ⟦book⟧ : Type
- Verbs are interpreted as predicates
  - ⟦walk⟧ : ⟦human⟧ → Prop
  - ⟦John walks⟧ = ⟦walk⟧ (j) : Prop

  where j = ⟦John⟧ : ⟦human⟧ .
- Adjectives are interpreted as predicates
  - ⟦handsome⟧ : ⟦man⟧ → Prop
  - ⟦handsome man⟧ = Σ(⟦man⟧ , ⟦handsome⟧)

Note: Many types in a modern type theory (e.g., Σ-types for modified CNs.)

Examples in Type-Theoretical Semantics

- What about, eg,
  - "A handsome man is a man" ?
  - "Paul walks", with p = ⟦Paul⟧ : ⟦handsome man⟧ ?
- Solution: coercive subtyping
  - First projection π_1 as coercion: $\Sigma(A,B) \leq_{\pi_1} A$
  - ⟦handsome man⟧ = Σ(**⟦man⟧** , **⟦handsome⟧**) $\leq_{\pi_1}$ **⟦man⟧**
  - ⟦Paul walks⟧ = ⟦walk⟧ (p) : Prop

    because

    ⟦walk⟧ : **⟦man⟧** → Prop and

    p : ⟦handsome man⟧ $\leq_{\pi_1}$ **⟦man⟧** .
- Remark:
  - Subtyping is crucial in type-theoretical semantics.

Example involving dot-types

- Recall
  - Example: PHY•INFO
  - PHY•INFO ≤ PHY and PHY•INFO ≤ INFO
- In type-theoretical semantics,

  ⟦book⟧ ≤ PHY•INFO

  ⟦boring⟧ : INFO → Prop ≤ PHY•INFO → Prop ≤ ⟦book⟧ → Prop

  So, ⟦boring book⟧ : Type

  ⟦burn⟧ : Human → PHY → Prop

  j = ⟦John⟧ : Man ≤ Human

  b : ⟦boring book⟧ ≤ ⟦book⟧ ≤ PHY•INFO ≤ PHY

  So, ⟦burn⟧ (j, b) : Prop

  So, ⟦John burned a boring book⟧

  = ∃ b : ⟦boring book⟧ . ⟦burn⟧ (j, b) : Prop

Example on Copredication

- Another example (copredication [Asher & Pustejovsky 05])

  "John picked up and mastered the book."

  ⟦pick up⟧ : ⟦human⟧ → PHY → Prop

  ≤ ⟦human⟧ → PHY•INFO → Prop

  ≤ ⟦human⟧ → ⟦book⟧ → Prop

  ⟦master⟧ : ⟦human⟧ → INFO → Prop

  ≤ ⟦human⟧ → PHY•INFO → Prop

  ≤ ⟦human⟧ → ⟦book⟧ → Prop
- Remark:
  - CNs as types in type-theoretical semantics – so things work.
  - Problematic if sticking to Montague's interpretations of CNs as functional subsets.

Modelling dot-types in type theory

- What is A•B?
  - Inadequate accounts (cf, [Asher 08]): Intersection types or product types
- Representing A•B as A×B with π_1/π_2 as coercions (formally as different type constructors):

$$\frac{A : \text{Type} \quad B : \text{Type} \quad C(A) \cap C(B) = \emptyset}{A \bullet B : \text{Type}}$$

Dot-types have two projections $p_1(a,b)=a$ and $p_2(a,b)=b$, both of which are coercions (see Luo 2010 for formal details).

Remark: The "C-condition" (disjointness of A/B-components) guarantees that the coercions concerned are coherent.

More subtyping for lexical semantics in TT

- Sense selection via overloading by coercive subtyping
  - Sense enumeration of homonymous words
  - Eg, John runs quickly.
    John runs a bank.
  - $[run]_1$: [human]→Prop
    $[run]_2$: [human]→[institution]→Prop
  - In general, let word w have different meanings $[w]_i : A_i$ (A_is are not equal or related by subtyping). Then, the sense enumeration model can be represented as coercions c_i:
    $c_i : 1_w \to A_i \qquad c_i(w) = [w]_i : A_i$
  - Then, correct senses are automated selected as expected.

Coercion contexts

- Reference transfers (Nunberg 1995)
- "The ham sandwich shouts"
- Coercion contexts with entries such as
  [ham sandwich] < [human]

Remark: we need coherent contexts.

Local coercions

- coercion $A <_c B$ in t
- Example of use: simultaneous use of
  the bank of the river
  the richest bank in the city
- More than one coercion from [bank]
  - Difficult to know which should be used.
  - They could even be incoherent!
- With local coercions, fine as expected.

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