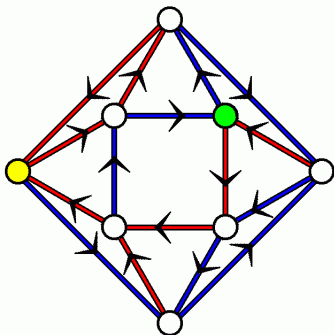


Buffered Communication Analysis in Distributed Multiparty Sessions

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Imperial College London



Asynchrony and buffer analysis

- Asynchronous communications rely on communication buffers, which may grow over time and overflow.
- Preventing communication buffers to overflow is a well-studied area.

Binary session types (two-party) [Takeuchi&al.'94]

- Binary session types specify channel behaviour in an asynchronous π -calculus (with message-order preserving, non-blocking sends).
- Binary session types give some information about buffers:

$$P_0 = s! \langle 3 \rangle . s! \langle \text{Orange} \rangle . s?(x) \quad T_0 = ! \langle \text{nat} \rangle ; ! \langle \text{string} \rangle ; ? \langle \text{real} \rangle ; \text{end}$$
$$P_1 = s! \langle 3 \rangle . s?(x) . s! \langle \text{Orange} \rangle \quad T_1 = ! \langle \text{nat} \rangle ; ? \langle \text{real} \rangle ; ! \langle \text{string} \rangle ; \text{end}$$

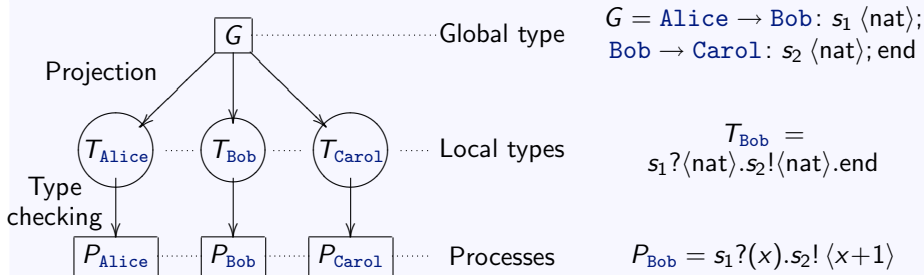
Multiparty sessions for protocol specification and analysis

- Today's distributed applications involve more and more agents that interact through complex communication patterns.
- Multiparty sessions types can describe these interactions and statically ensure type and communication safety and fidelity to a stipulated protocol.

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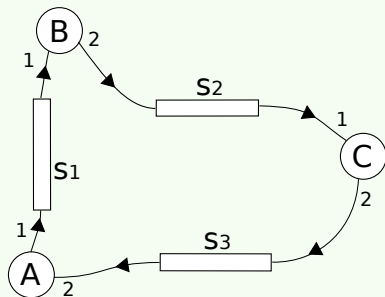
Multiparty session types in a nutshell



Buffer analysis in the multiparty case

Binary session type techniques to analyse buffer usage do not apply directly.

Example (a)



(A) $\text{Alice} = \mu X. s_1! \langle 1 \rangle; s_3?(x); X$

(B) $\text{Bob} = \mu X. s_1?(x); s_2! \langle \text{Orange} \rangle; X$

(C) $\text{Carol} = \mu X. s_2?(x); s_3! \langle 2.4 \rangle; X$

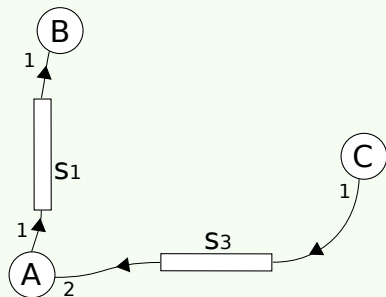
$$G_a = \mu x. \text{Alice} \rightarrow \text{Bob}: s_1 \langle \text{nat} \rangle;$$
$$\text{Bob} \rightarrow \text{Carol}: s_2 \langle \text{string} \rangle;$$
$$\text{Carol} \rightarrow \text{Alice}: s_3 \langle \text{real} \rangle; x$$

- All three buffers need not hold more than one value.

Buffer analysis in the multiparty case

Binary session type techniques to analyse buffer usage do not apply directly.

Example (b)



(A) $Alice = \mu X. s_1! \langle 1 \rangle; s_3?(x); X$

(B) $Bob = \mu X. s_1?(x); X$

(C) $Carol = \mu X. s_3! \langle 2.4 \rangle; X$

$G_b = \mu x. Alice \rightarrow Bob: s_1 \langle nat \rangle;$

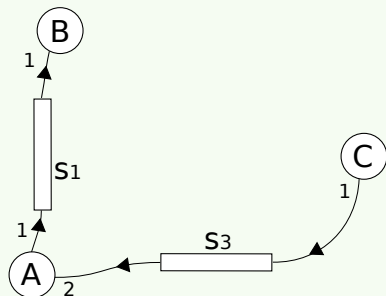
$Carol \rightarrow Alice: s_3 \langle real \rangle; x$

- Values can accumulate in the two remaining buffers.

Buffer analysis in the multiparty case

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Example (b)



(A) $Alice = \mu X. s_1! \langle 1 \rangle; s_3?(x); X$

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$G_b = \mu x. Alice \rightarrow Bob: s_1 \langle nat \rangle;$

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- Values can accumulate in the two remaining buffers.
- **Alice's** local behaviour is the same in example (a) and (b).

- I A general causality analysis of global session types
 - a An algorithm to detect infinite buffers
 - b An algorithm to compute the buffer size of finite buffers
 - c A type system to ensure communication, buffer and type safety and progress properties.
- II A framework for session programming and analysis
 - a Channel allocation
 - b Global refinement
 - c Local refinement (messaging optimisation)

- | A general causality analysis of global session types
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 - c A type system to ensure communication, buffer and type safety and progress properties.

Global types and session graphs

Syntax of global types

$$\begin{array}{l} G ::= p \rightarrow p' : k \langle U \rangle ; G' \quad \text{values} \\ \quad | \quad p \rightarrow p' : k \{ l_j : G_j \}_{j \in J} \quad \text{branching} \\ \quad | \quad \mu x. G \quad | \quad x \quad | \quad \text{end} \quad \text{recursion, end} \end{array}$$

Running example

$$\begin{array}{l} \mu x. \text{Alice} \rightarrow \text{Bob} : s_1 \langle \text{nat} \rangle ; \text{Bob} \rightarrow \text{Carol} : s_2 \\ \quad \left\{ \begin{array}{l} l_1 : \text{Carol} \rightarrow \text{Alice} : s_3 \{ l_3 : \text{Alice} \rightarrow \text{Carol} : s_2 \langle \text{string} \rangle ; x \} \\ l_2 : \text{Carol} \rightarrow \text{Alice} : s_3 \{ l_4 : \text{end} \} \end{array} \right\} \end{array}$$

Global types and session graphs

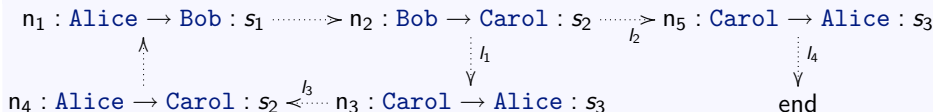
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Session graph



Global types and session graphs

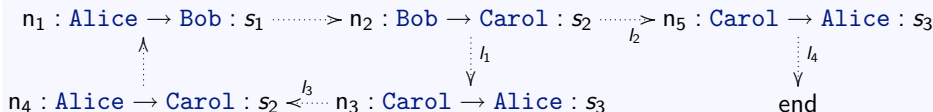
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Session graph



- Causality analysis = determining which edges stand up to asynchrony

Causality Analysis and Unbounded Buffers

- An IO-dependency links two communications when one guards the other.

Alice \rightarrow Bob: s_1 \langle nat \rangle ;

Bob \rightarrow Carol: s_2 \langle string \rangle ;

- IO-chains guarantee that a buffer is read before proceeding.

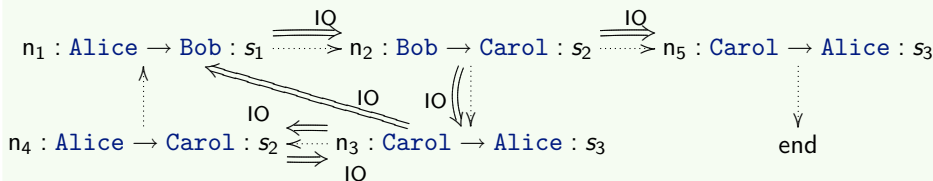
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IO-dependencies on a session graph



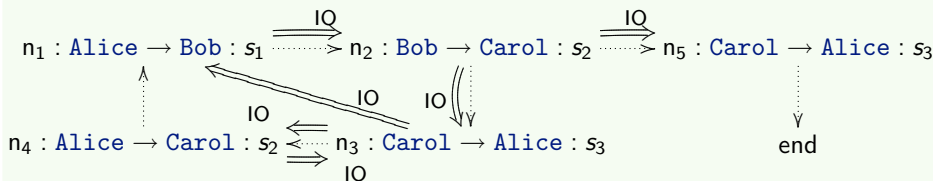
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IO-dependencies on a session graph



Infinite buffers

If a channel is used in a graph cycle that does not contain an IO-cycle, then this channel is infinite.

Reset Property

The Reset property characterises the paths in the session graph that see a buffer being emptied.

Reset property

A path satisfies the Reset property if its last node is the end of an IO-chain that starts with a node using the same buffer.

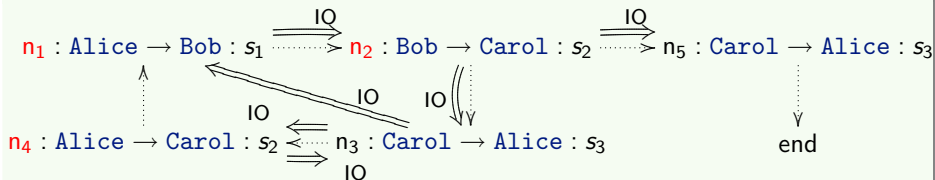
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Example



$\text{Reset}(n_4 n_1 n_2)$

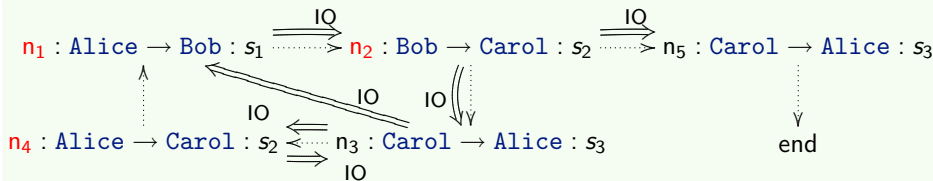
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Example



$\text{Reset}(n_4 n_1 n_2)$ does not hold,

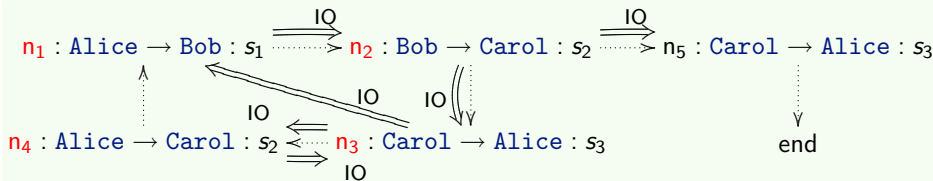
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Example



$\text{Reset}(n_4 n_1 n_2)$ does not hold, $\text{Reset}(n_1 n_2 n_3 n_4)$

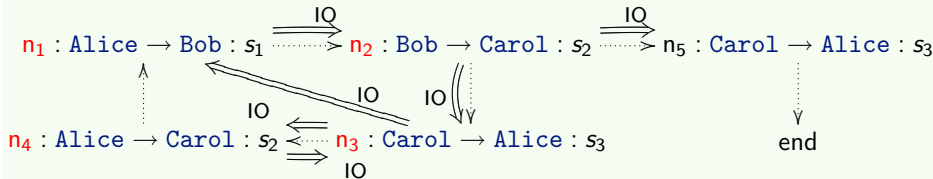
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Example



$\text{Reset}(n_4 n_1 n_2)$ does not hold, $\text{Reset}(n_1 n_2 n_3 n_4)$ holds.

An algorithm to compute buffer sizes

- An edge in a session graph between nodes n_1 and n_2 is noted $n_1 \prec n_2$
- All values are considered of size 1.

Algorithm (computes the bound for finite channel k)

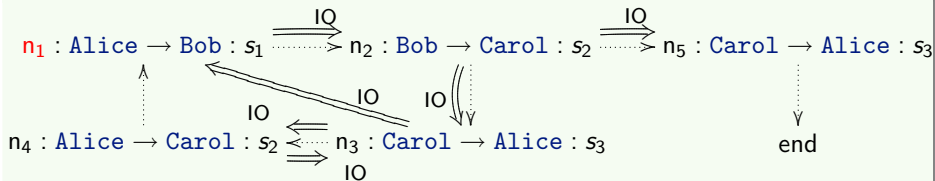
$$\mathcal{B}_k \langle m, \mathcal{P}, \tilde{n}, n \rangle = \begin{cases} 0 & \text{if } n = \text{end or } \tilde{n} \in \mathcal{P} \\ \max_{n \prec n'} \mathcal{B}_k \langle m, \{\tilde{n}\} \cup \mathcal{P}, \tilde{n}n, n' \rangle & \text{if } \text{ch}(n) = k', k \neq k' \\ \max_{n \prec n'} \mathcal{B}_k \langle 1, \{\tilde{n}\} \cup \mathcal{P}, n, n' \rangle & \text{if } \text{ch}(n) = k, \text{Reset}(\tilde{n}n) \\ \max(m + 1, \max_{n \prec n'} \mathcal{B}_k \langle m + 1, \{\tilde{n}\} \cup \mathcal{P}, \tilde{n}n, n' \rangle) & \text{if } \text{ch}(n) = k, \neg \text{Reset}(\tilde{n}n) \end{cases}$$

- $\text{Reset}(\tilde{n})$ can be computed incrementally
- the length of paths is bounded by linearity and the number of participants

This gives an efficient and practical algorithm.

Example of buffer bound computation

Example: bound of channel s_2



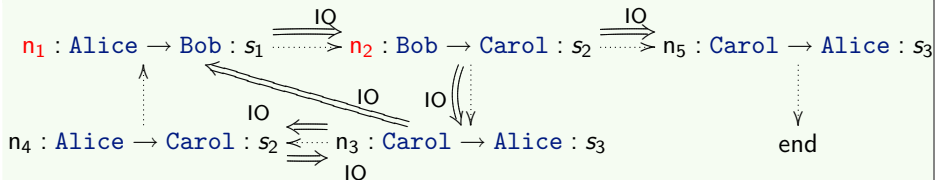
$B_{s_2} \langle 0, \epsilon, n_1 \rangle$

max

explanation

Example of buffer bound computation

Example: bound of channel s_2

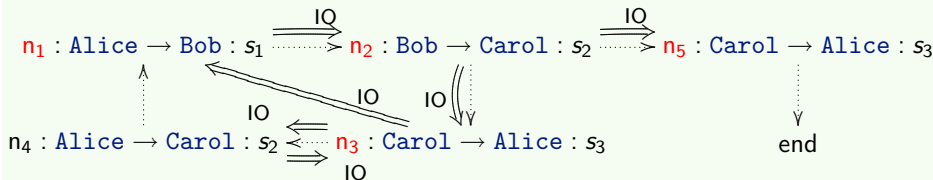


$$\begin{aligned} & \mathcal{B}_{s_2} \langle 0, \epsilon, n_1 \rangle \\ &= \mathcal{B}_{s_2} \langle 0, n_1, n_2 \rangle \end{aligned}$$

max	explanation
0	$s_1 \neq s_2$

Example of buffer bound computation

Example: bound of channel s_2

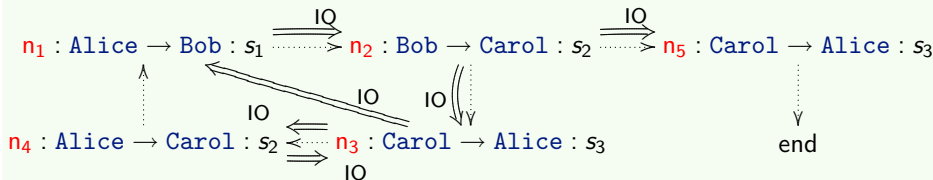


$$\begin{aligned}
 & \mathcal{B}_{s_2} \langle 0, \epsilon, n_1 \rangle \\
 = & \mathcal{B}_{s_2} \langle 0, n_1, n_2 \rangle \\
 = & \max(\mathcal{B}_{s_2} \langle 1, n_1 n_2, n_3 \rangle, \mathcal{B}_{s_2} \langle 1, n_1 n_2, n_5 \rangle)
 \end{aligned}$$

max	explanation
0	$s_1 \neq s_2$
1	$\neg \text{Reset}(n_1 n_2)$

Example of buffer bound computation

Example: bound of channel s_2

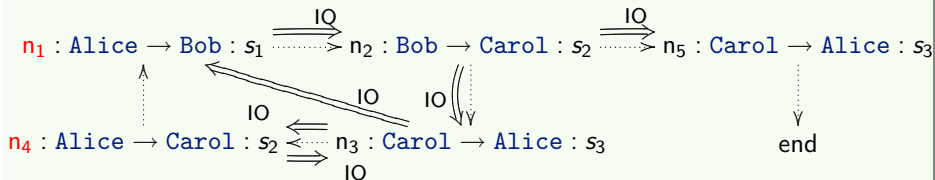


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1	$s_3 \neq s_2$

Example of buffer bound computation

Example: bound of channel s_2

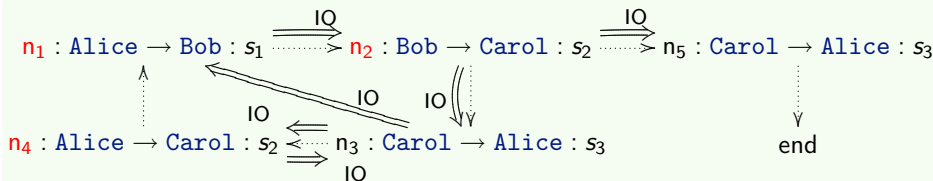


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 = & \max(\mathcal{B}_{s_2} \langle 1, n_4, n_1 \rangle, 0)
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max	explanation
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Example of buffer bound computation

Example: bound of channel s_2

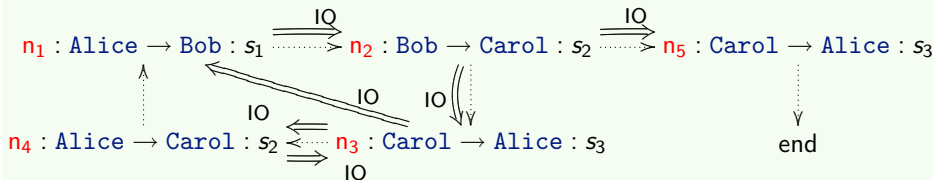


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Example of buffer bound computation

Example: bound of channel s_2

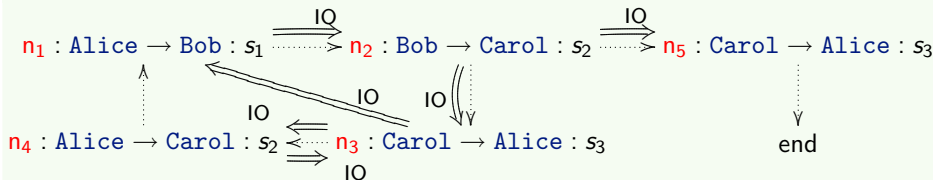


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max	explanation
0	$s_1 \neq s_2$
1	$\neg \text{Reset}(n_1 n_2)$
1	$s_3 \neq s_2$
1	$\text{Reset}(n_1 n_2 n_3 n_4)$
1	$s_1 \neq s_2$
2	$\neg \text{Reset}(n_4 n_1 n_2)$

Example of buffer bound computation

Example: bound of channel s_2

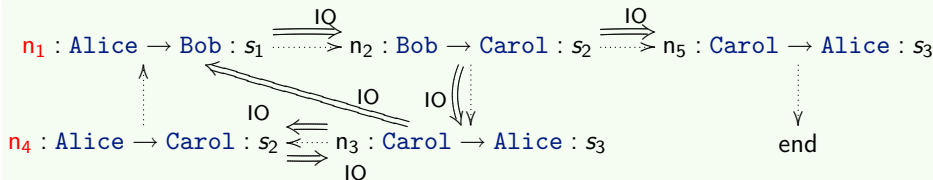


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Example of buffer bound computation

Example: bound of channel s_2



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2	$s_3 \neq s_2$
2	$\text{Reset}(n_4 n_1 n_2 n_3 n_4)$

Processes, semantics and type system

Thanks to this algorithm, we can statically enforce buffer safety by typing.

Processes and semantics

$$\begin{aligned} & \bar{a}[2..n](\tilde{s}^{\tilde{n}}).P_1 \\ & \quad | a[2](\tilde{s}).P_2 \mid \dots \mid a[n](\tilde{s}).P_n \rightarrow (\nu \tilde{s})(P_1 \mid P_2 \mid \dots \mid P_n \mid s_1^{n_1}:\emptyset \mid \dots \mid s_m^{n_m}:\emptyset) \\ & \quad s!\langle e \rangle; P \mid s^n:\tilde{h} \rightarrow P \mid s^n:\tilde{h} \cdot v \quad (n \geq |\tilde{h}|, e \downarrow v) \\ & \quad s?(x); P \mid s^n:v \cdot \tilde{h} \rightarrow P[v/x] \mid s^n:\tilde{h} \end{aligned}$$

Typing system

$$\frac{\Gamma \vdash a : G \quad \Gamma \vdash P \triangleright \Delta, \tilde{s}^{\tilde{m}} : (G \upharpoonright 1) \quad |\tilde{s}| = \text{chan}(G) \quad \mathcal{B}_k \langle G \rangle = m_k}{\Gamma \vdash \bar{a}[2..n](\tilde{s}^{\tilde{m}}).P \triangleright \Delta}$$

Theorem (Subject reduction)

$\Gamma \vdash_{\Sigma} P \triangleright \Delta$ and $P \longrightarrow Q$ with $(\Gamma, \Sigma, P, \Delta)$ fully coherent imply $\Gamma \vdash_{\Sigma} Q \triangleright \Delta'$ for some Δ' , s_k such that $\Delta = \Delta'$ or $\Delta \xrightarrow{(s_k)^*} \Delta'$ and $\mathcal{B}\langle G(k) \rangle \geq \mathcal{B}\langle G'(k) \rangle$ with $\Delta(\check{s}) = \llbracket G \rrbracket$, $\Delta'(\check{s}) = \llbracket G' \rrbracket$ and $(\Gamma, \Sigma, Q, \Delta')$ fully coherent.

We define the *buffer overflow error* as follows:

$$n \leq |\tilde{h}| \quad \Rightarrow \quad s! \langle \tilde{e} \rangle; P \mid s^n : \tilde{h} \rightarrow \text{Err}$$

Corollary (Buffer safety)

If $\Gamma \vdash_{\Sigma} P \triangleright \Delta$, then for all P' s.t. $P \longrightarrow^* P'$, $P' \not\rightarrow \text{Err}$.

We can also derive using standard techniques:

- Communication safety
- Progress

- I A general causality analysis of global session types
 - a An algorithm to detect infinite buffers
 - b An algorithm to compute the buffer size of finite buffers
 - c A type system to ensure communication, buffer and type safety and progress properties.
- II A framework for session programming and analysis
 - a Channel allocation
 - b Global refinement
 - c Local refinement (messaging optimisation)

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Channel Allocation (by example)

When designing a session, our buffer analysis can help allocate channels

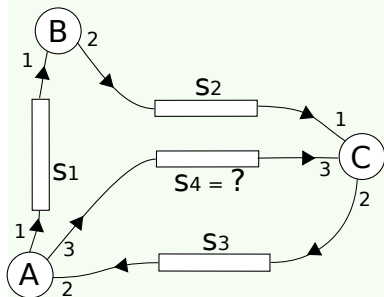
- with constraints such as a maximum number of channels or
- a maximum size of channels

Channel Allocation (by example)

When designing a session, our buffer analysis can help allocate channels

- with constrains such as a maximum number of channels or
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Example (c)



(A₂) $Alice_2 = \mu X. s_1! \langle 1 \rangle; s_3?(x); s_4! \langle x+1 \rangle; X$

(B) $Bob = \mu X. s_1?(x); s_2! \langle Orange \rangle; X$

(C₂) $Carol_2 = \mu X. s_2?(x); s_3! \langle 2.4 \rangle; s_4?(y); X$

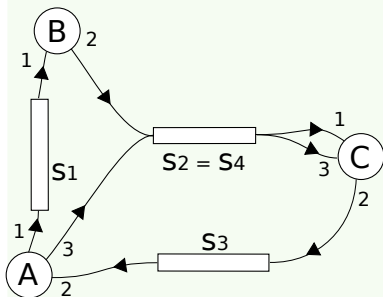
$G_c = \mu x. Alice \rightarrow Bob: s_1 \langle nat \rangle;$
 $Bob \rightarrow Carol: s_2 \langle string \rangle;$
 $Carol \rightarrow Alice: s_3 \langle real \rangle;$
 $Alice \rightarrow Carol: s_4 \langle nat \rangle; x$

Channel Allocation (by example)

When designing a session, our buffer analysis can help allocate channels

- with constrains such as a maximum number of channels or
- a maximum size of channels

Example (d)



(A₃) $Alice_3 = \mu X. s_1! \langle 1 \rangle; s_3?(x); s_2! \langle x+1 \rangle; X$

(B) $Bob = \mu X. s_1?(x); s_2! \langle Orange \rangle; X$

(C₃) $Carol_3 = \mu X. s_2?(x); s_3! \langle 2.4 \rangle; s_2?(y); X$

$G_d = \mu x. Alice \rightarrow Bob: s_1 \langle nat \rangle;$
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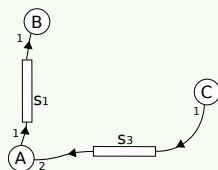
Global refinement (by example)

How do we fix overflowing buffers?

- when there is an infinite buffer or
- when a buffer size does not occupy the available space

We can insert *confirmation* (synchronisation, acknowledgement) messages

We set buffers to be of size at most 2


$$G_b = \mu x. \text{Alice} \rightarrow \text{Bob}: s_1 \langle \text{nat} \rangle; \text{Carol} \rightarrow \text{Alice}: s_3 \langle \text{real} \rangle; x$$

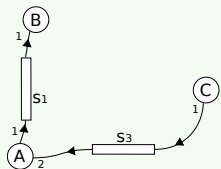
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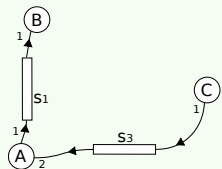
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Messaging optimisation by example

$$P_1 = \mathbf{t?}(y); \mathbf{s!} \langle \mathbf{5} \rangle; \mathbf{s!} \langle \text{Orange} \rangle$$
$$P_2 = \mathbf{s?}(z_1); \mathbf{s?}(z_2)$$
$$P_3 = \mathbf{b?}(y); \mathbf{t!} \langle \mathbf{7} \rangle$$

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Messaging optimisation by example

$P_1 = t?(y); s! \langle 5 \rangle; s! \langle \text{Orange} \rangle$

$P_2 = s?(z_1); s?(z_2)$

$P_3 = b?(y); t! \langle 7 \rangle$

$b?(y)$ blocks the communication.

Messaging optimisation by example

$$P'_1 = s! \langle 5 \rangle; s! \langle \text{Orange} \rangle; t?(y)$$
$$P_2 = s?(z_1); s?(z_2)$$
$$P'_3 = t! \langle 7 \rangle; b?(y)$$

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Messaging optimisation by example

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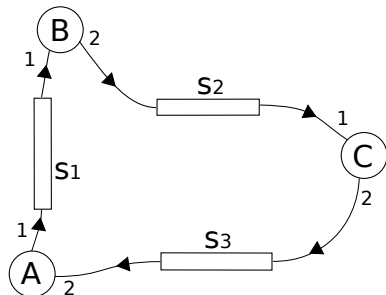
$$P'_3 = t! \langle 7 \rangle; b?(y)$$

$b?(y)$ blocks the communication.

Communication subtyping [ESOP'09]

$$! \langle U \rangle; ? \langle T \rangle \leq ? \langle T \rangle; ! \langle U \rangle$$

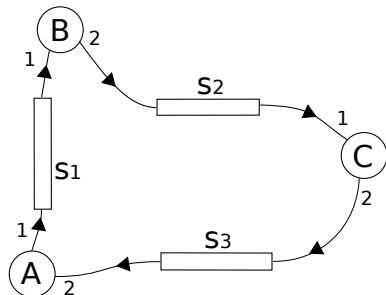
Local refinement does not preserve buffer sizes



$G = \mu x. \text{Alice} \rightarrow \text{Bob}: s_1 \langle \text{nat} \rangle;$
 $\text{Bob} \rightarrow \text{Carol}: s_2 \langle \text{string} \rangle;$
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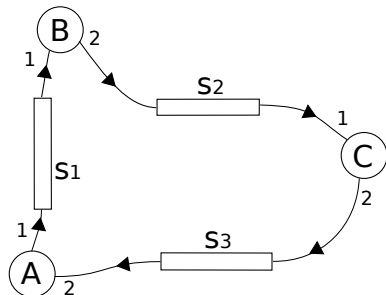
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- Local refinement can potentially change any buffer requirement.
- How can we determine which optimisations are safe?

Acceptable local refinement

We define a new subtyping relation that respect established buffer bounds.

Dependent nodes of a channel k

- Reset nodes: nodes that are part of an IO-chain that resets a path
- Minimal resetting paths:
Paths that satisfy the reset property while none of their suffix does
- Dependent nodes $\text{dep}(k)$:
Union of the reset nodes of the minimal resetting paths that end with k

Dependent nodes are guards of a communication and control buffer sizes.

For a global type G , we choose a partitioning $\{N_0, \dots, N_n\}$ of the set of nodes of G . This partition should satisfy the two properties:

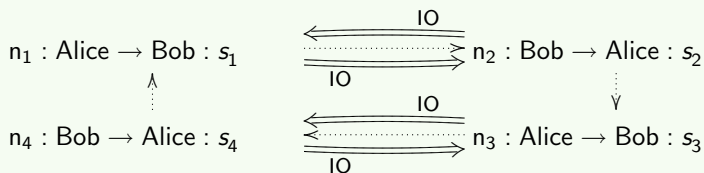
$\forall n \in N_i, n' \in N_j, \text{ch}(n) = \text{ch}(n') \Rightarrow N_i = N_j$ and $\forall n \in N_i, \text{dep}(\text{ch}(n)) \subset N_i$.

Asynchronous subtyping

$$(OI) \quad k^N! \langle U \rangle; k_0^{N_0}? \langle U' \rangle; T \ll k_0^{N_0}? \langle U' \rangle; k^N! \langle U \rangle; T \quad (N \cap N_0 = \emptyset)$$

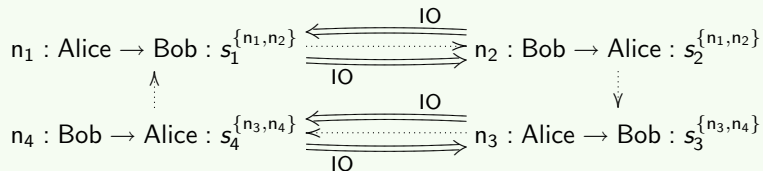
Examples of local refinement

Annotation of session channels and messaging optimisation



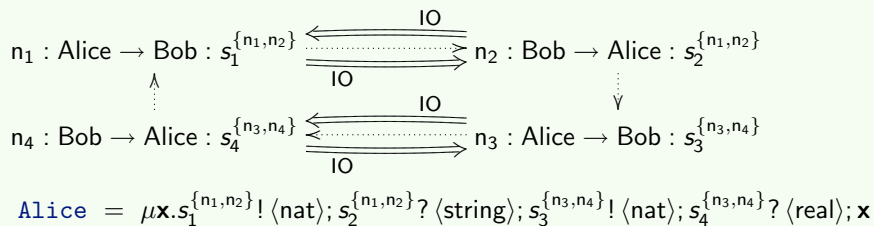
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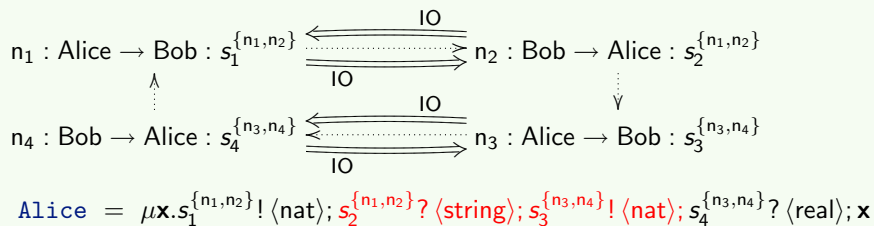
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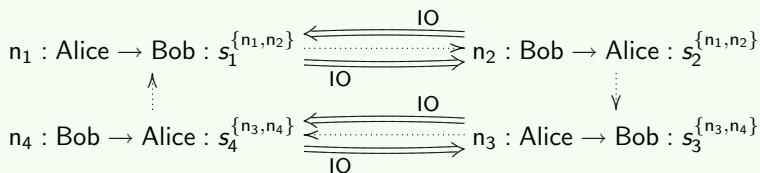
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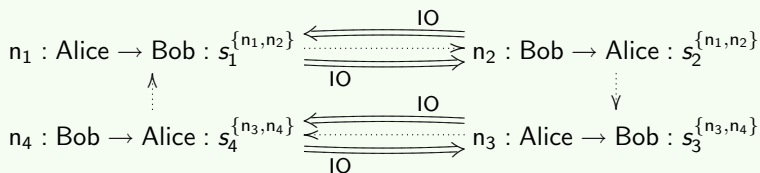
Annotation of session channels and messaging optimisation



$\text{Alice} = \mu \mathbf{x}. s_1^{\{n_1, n_2\}} ! \langle \text{nat} \rangle ; s_3^{\{n_3, n_4\}} ! \langle \text{nat} \rangle ; s_2^{\{n_1, n_2\}} ? \langle \text{string} \rangle ; s_4^{\{n_3, n_4\}} ? \langle \text{real} \rangle ; \mathbf{x}$

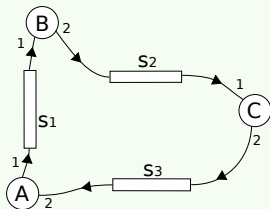
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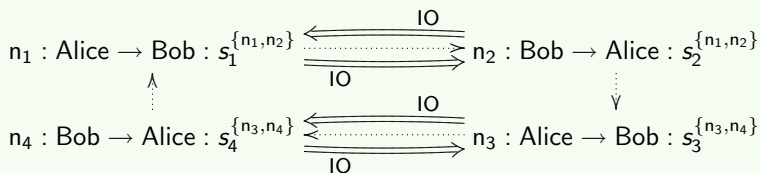
Unsafe optimisation



$G = \mu x. \text{Alice} \rightarrow \text{Bob} : s_1 \langle \text{nat} \rangle;$
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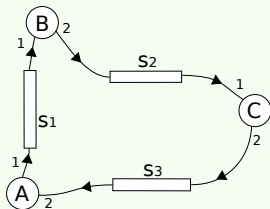
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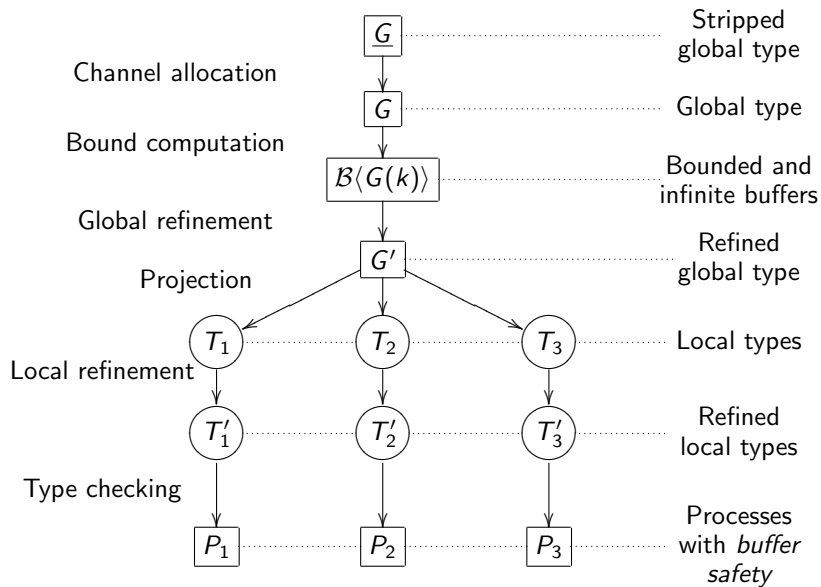
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Only the trivial partition is possible!

Overview

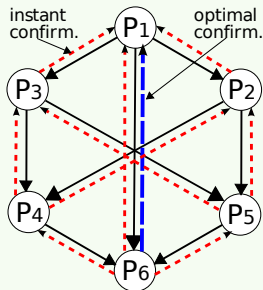


- Multi-buffering algorithm [ESOP'09] which used unsafe optimisation.

Application

- Multi-buffering algorithm [ESOP'09] which used unsafe optimisation.

Multiprocessor SoC [Cheung&al. HLDTV'07]



Six processes connected by nine FIFOs

$$G = \mu x. (\text{foreach}(i \in \{2, 3\})\{ G[i]; p[1] \rightarrow p[6] : \text{on} \}; x$$
$$G[i] = p[1] \rightarrow p[i] : \begin{cases} \text{on} : p[i] \rightarrow p[4] : \text{on}; p[4] \rightarrow p[6] : \text{on}; \\ \text{off} : p[i] \rightarrow p[5] : \text{off}; p[5] \rightarrow p[6] : \text{off} \end{cases}$$

Proposition

The above MPSoC six processes are communication-safe, buffer safe and satisfy progress (after global and local refinements).

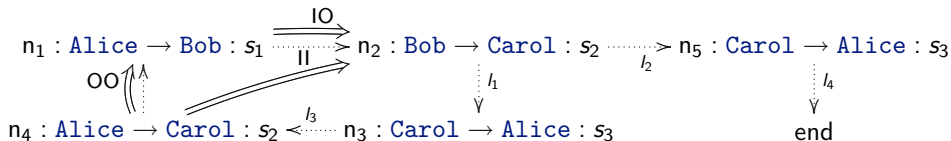
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Thanks!

- Linearity

Causality Analysis and Linearity



- OO-dependency: local sends are ordered.
- II-dependency: local receives are ordered.
- IO-dependency: reception is a guard for sends.
- Linearity property: two uses of the same channels cannot be in conflict
The senders should be ordered by OO-chains and the receivers by II-chains.