Formal Semantics in Modern Type Theories
An Overview*

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I'll give an overview, and report some recent developments, of Formal Semantics in Modern Type Theories (MTT-semantics for short) [25, 14, 4]. MTT-semantics is a semantic framework for natural language, in the tradition of Montague's semantics [21]. However, while Montague's semantics is based on Church's simple type theory [5, 8] (and its models in set theory), MTT-semantics is based on dependent type theories, which we call modern type theories (MTTs),[1] to distinguish them from the simple type theory. Thanks to the recent development, MTT-semantics has become not only a full-blown alternative to Montague's semantics, but also a very attractive framework with a promising future for linguistic semantics.

In this talk, MTT-semantics will be explicated, and its advantages explained, by focussing on the following:

1. The rich structures in MTTs, together with subtyping, make MTTs a nice and powerful framework for formal semantics of natural language.

2. MTT-semantics is both model-theoretic and proof-theoretic and hence very attractive, both theoretically and practically.

By explaining the first point, we'll introduce MTT-semantics and, at the same time, show that the use and development of subtyping [13, 17] play a crucial role in making MTT-semantics viable. The second point, based on [15, 16, 11, 4], shows that MTTs provide a unique and nice semantic framework that was not available before for linguistic semantics. Being model-theoretic, MTT-semantics provides a wide coverage of various linguistic features and, being proof-theoretic, its foundational languages have proof-theoretic meaning theory based on inferential uses[2] (appealing philosophically and theoretically) and it establishes a solid foundation for practical reasoning in natural languages on proof assistants such as Coq [3] (appealing practically). Altogether, this strengthens the argument that MTT-semantics is a promising framework for formal semantics, both theoretically and practically.

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[1] By MTTs, we refer to the family of formal systems such as Martin-Löf's intensional type theory (MLTT) [18, 22] in Agda, the type theory CIC_p in Coq [6] and the Unifying Theory of dependent Types (UTT) [12] in Lego/Plastic.

[2] Proof-theoretic semantics, in the sense of [10], has been studied by logicians such as Gentzen [9], Prawitz [24, 23] and Martin-Löf [19, 20] and discussed by philosophers such as Dummett [7] and Brandom [1, 2], among others.

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