Adjectives in a Modern Type-Theoretical Setting*

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Abstract. In this paper we discuss the semantics of adjectives from the perspective of a Modern Type Theory (MTT) with an adequate subtyping mechanism. In an MTT, common nouns (CNs) can be interpreted as types and, in particular, CNs modified by intersective and subsective adjectives can be given semantics by means of Σ-types. However, an interpretation of CNs as types would not be viable without a proper notion of subtyping which, as we explain, is given by coercive subtyping, an adequate notion of subtyping for MTTs. It is also shown that suitable uses of universes are one of the key ingredients that have made such an analysis adequate. Privative and non-committal adjectives require different treatments than the use of Σ-types. We propose to deal with privative adjectives using the disjoint union type while non-committal adjectives by making use of the type-theoretical notion of context, as used by Ranta [27] to approximate the model-theoretic notion of a possible world. Our approach to adjectives has a number of advantages over those proposed within the Montagovian setting, one of which is that the inferences related with the adjectives arise via typing and not by some kind of extra semantic meaning in the form of a meaning postulate.

1 Introduction

The semantics of adjectives is a well-studied issue in the Montagovian tradition and a number of proposals have been put forth by the years (see e.g.[22],[10] [24] and [25] among others). Another prominent line of research on adjectives is based on Davidson's [6] treatment of adverbials and adjectives. In these approaches, the semantics of adverbial and adjectival modification are derived by exploiting the additional event argument assumed in Davidsonian semantics (see for example [11]). In modern Type Theories (MTTs), i.e. TTs within the tradition of Martin-Löf, adjectival modification has been treated as a Σ-type (see e.g. [27], [15]). However, such an approach in the form it is proposed (e.g. as in [27]) can only

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deal with subsective adjectives. The intersective adjectival class can be treated with Σ-types, but as we shall see, not in the way proposed in the literature.

In this paper, we discuss the issue of adjectival modification within an MTT equipped with an adequate subtyping mechanism (coercive subtyping). We first show that a Σ-type analysis can accommodate both intersective and subsective adjectives. This is due to the subtyping mechanism as well as the use of the universe CNof (the interpretations of) common nouns for the cases where these are needed.[3] In order to deal with the privative class, we propose to treat privative adjectival modification via disjoint union types. Such a move is quite close (at least on a pre-theoretical level) to Partee's treatment of adjectives like *fake* as being subsective but applied to CNs with coerced meanings [25]. Lastly, the case of non-committal adjectives is discussed arguing that one can have an adequate MTT account by exploiting the constructive notion of context.

The paper is structured as follows: in §2, we introduce the framework to be used, concentrating on the features that are relevant for the treatment of adjectives. Starting from §3 to §5, we shall study intersective/subsective, privative and non-committal adjectives, respectively. In §3 we consider the Σ-type analysis of CNs modified by intersective and subsective adjectives and discuss how subsective adjectives should be dealt with in such a context. In §4, by further developing a proposal by the second author [19], we study how privative adjectives may be interpreted by means of disjoint union types together with coercive subtyping. Non-committal adjectives are studied in §5, where it is shown that they may be given semantics by considering Ranta's formulation of belief contexts [27]. Finally, in §6, adjectives like *former* are briefly studied as special temporal cases, while we further make a proposal on how time may be incorporated by means of dependent types.

2 An MTT with Coercive Subtyping

In this section, we give a brief introduction to the formal semantics based on Modern Type Theories (MTTs) [27, 14, 17]. A Modern Type Theory (MTT) is a variant of a class of type theories as studied by Martin-Löf [20, 21] and others, which have dependent types and inductive types, among others. We choose to call them Modern Type Theories in order to distinguish them from Church's simple type theory [5] that is commonly employed within the Montagovian tradition in formal semantics.

Among the variants of MTTs, we are going to employ the Unified Theory of dependent Types (UTT) [12] with the addition of the coercive subtyping mechanism (see, for example, [13, 18] and below). UTT is an impredicative type theory in which a type of all logical propositions ($Prop$) exists.[4] This stands as part of the study of linguistic semantics using MTTs rather than simple typed ones, including the early studies such as [28, 27] inter alios.

[3] See §2.4 for the notion of a universe.

[4] This is similar to the simple type theory where there is a type t of truth values.

	Example	Montague semantics	MTT-based Semantics
CN	man, human	$[\![man]\!],\ [\![human]\!] : e \to t$	$[\![man]\!],\ [\![human]\!] : Type$
IV	talk	$[\![talk]\!] : e \to t$	$[\![talk]\!] : [\![human]\!] \to Prop$
ADJ	handsome	$[\![handsome]\!] : (e \to t) \to (e \to t)$	$[\![handsome]\!] : [\![man]\!] \to Prop$
MCN	handsome man	$[\![handsome]\!]([\![man]\!]) : e \to t$	$\Sigma m : [\![man]\!] \,.\, [\![handsome]\!](m) : Type$
S	A man talks	$\exists m : e.\ [\![man]\!](m) \& [\![talk]\!](m) : t$	$\exists m : [\![man]\!] \,.\, [\![talk]\!](m) : Prop$

Fig. 1. Examples in formal semantics.

2.1 Formal Semantics Based on MTTs: the Basics

In MTT-based semantics, the basic ways to interpret various linguistic categories is as follows:[5]

- A sentence (S) is interpreted as a proposition of type *Prop*.
- A common noun (CN) can be interpreted as a type.
- A verb (IV) can be interpreted as a predicate over the type D that interprets the domain of the verb (ie, a function of type $D \to Prop$).
- An adjective (ADJ) can be interpreted as a predicate over the type that interprets the domain of the adjective (ie, a function of type $D \to Prop$).
- Modified common nouns (MCNs) can be interpreted by means of Σ-types (see below).

In what follows, we shall give further explanations of various aspects of semantics based on MTTs, explicating along the way the basic features of MTTs and coercive subtyping. We try to bring out the linguistic relevance of the system used rather than being meticulous as regards the formal details in each case.

2.2 Common Nouns as Types and Many-sortedness of MTTs

A key difference between the formal semantics based on MTTs on the one hand and Montague semantics on the other, lies in the interpretation of common nouns (CNs). This is in turn based on the fact that MTTs are essentially 'many-sorted' logical systems.

In Montague semantics [23], the underlying logic (Church's simple type theory [5]) can be seen as 'single-sorted' in the sense that there is only one type e of all entities. The other types such as t of truth values and the function types generated from e and t do not stand for types of entities. In this respect, there are no fine-grained distinctions between the elements of type e and as such all individuals are interpreted using the same type. For example, *John* and *Mary* have the same type in simple type theories, the type e of individuals. An MTT, on the other hand, can be regarded as a 'many-sorted' logical system in that it contains many types and. In this respect, in an MTT-based semantics one can

[5] Basic examples are shown in Figure 1, along with a comparison with their counterparts in Montague semantics

make fine-grained distinctions between individuals and use those different types to interpret subclasses of individuals. For example, we can have $John : [\![man]\!]$ and $Mary : [\![woman]\!]$, where $[\![man]\!]$ and $[\![woman]\!]$ are different types.

An important trait of MTT-based semantics is the interpretation of common nouns (CNs) as *types* [27] rather than sets or predicates (i.e., objects of type $e \to t$) as in Montague semantics. The CNs man, human, table and book are interpreted as types $[\![man]\!]$, $[\![human]\!]$, $[\![table]\!]$ and $[\![book]\!]$, respectively. Then, individuals are interpreted as being of one of the types used to interpret CNs.

Modified common nouns (MCNs in Figure 1) can be interpreted by means of Σ-types, types of dependent pairs. For instance, 'handsome man' can be interpreted as the type $\Sigma m : [\![man]\!] .\ [\![handsome]\!](m)$, the type of pairs of a man and a proof that the man is handsome.

This many-sortedness (i.e., the fact that there are many types in an MTT) has the welcoming result that a number of semantically infelicitous sentences like e.g. *the ham sandwich walks*, which are however syntactically well-formed, can be explained easily given that a verb like walks will be specified as being of type $Animal \to Prop$ while the type for *ham sandwich* will be $[\![food]\!]$ or $[\![sandwich]\!]$, which is not compatible with the typing for *walks*:[6]

(1) $the\ ham\ sandwich : [\![food]\!]$

(2) $walk : [\![human]\!] \to Prop$

The idea of common nouns being interpreted as types rather than predicates has been argued in [16] on philosophical grounds as well. There, the author argues that Geach's observation that common nouns, in contrast to other linguistic categories, have criteria of identity that enable common nouns to be compared, counted or quantified, has an interesting link with the constructive notion of set/type: in constructive mathematics, sets (types) are not constructed only by specifying their objects but they additionally involve an equality relation. The argument is then that the interpretation of CNs as types in MTTs is explained and justified to a certain extent.[7]

Interpreting CNs as types rather than predicates has also a significant methodological implication: this is compatible with various subtyping relations one may consider in formal semantics. For instance, in modelling some linguistic phenomena semantically, one may introduce various subtyping relations by postulating a collection of subtypes (physical objects, informational objects, eventualities, etc.) of the type of entities [1]. It has become clear that, if CNs are interpreted as predicates as in the traditional Montagovian setting, introducing such subtyping relations would cause difficult problems: even some basic semantic interpretations would go wrong and it is very difficult to deal with some linguistic phenomena such as copredication satisfactorily. Instead, if CNs are interpreted as types, as in the type-theoretical semantics based on MTTs, copredication can be given a straightforward and satisfactory treatment [14].

[6] This is of course based on the assumption that the definite NP is of a lower type and not a Generalized Quantifier.

[7] See [16] for more details on this.

2.3 Subtyping in Formal Semantics

As briefly explained above, because of many-sortedness of MTTs, CNs can be interpreted as types. For instance, in a Montagovian setting, all of the verbs below are given the same type $e \to t$, but in an MTT, we can have

(3) $drive : [\![human]\!] \to Prop$

(4) $eat : [\![animal]\!] \to Prop$

(5) $disappear : [\![object]\!] \to Prop$

which have different domain types. This has the advantage of disallowing interpretations of some infelicitous examples like *the ham sandwich walks*.

However, interpreting CNs by means of different types could lead to serious undergeneralizations without a subtyping mechanism: *subtyping* is crucial for an MTT-based semantics. For instance, consider the interpretation of the sentence 'A man talks' in Figure 1: for m of type $[\![man]\!]$ and $[\![talk]\!]$ of type $[\![human]\!] \to Prop$, the function application $[\![talk]\!](m)$ is only well-typed because we have that $[\![man]\!]$ is a subtype of $[\![human]\!]$.

Coercive subtyping [13, 18] provides an adequate framework to be employed for MTT-based formal semantics [14, 17].[8] It can be seen as an abbreviation mechanism: A is a (proper) subtype of B ($A < B$) if there is a unique implicit coercion c from type A to type B and, if so, an object a of type A can be used in any context $\mathfrak{C}_B[\_]$ that expects an object of type B: $\mathfrak{C}_B[a]$ is legal (well-typed) and equal to $\mathfrak{C}_B[c(a)]$.

As an example, in the case that both $[\![man]\!]$ and $[\![human]\!]$ are base types, one may introduce the following as a basic subtyping relation:

(6) $[\![man]\!] \ < \ [\![human]\!]$

In case that $[\![man]\!]$ is defined as a composite Σ-type (see §2.4 below for details), where $male : [\![human]\!] \to Prop$:

(7) $[\![man]\!] = \Sigma h : [\![human]\!] \,.\, male(h)$

we have that (6) is the case because the above Σ-type is a subtype of $[\![human]\!]$ via the first projection π_1:

(8) $(\Sigma h : [\![human]\!] \,.\, male(h)) <_{\pi_1} \ [\![human]\!]$

Equipped with this coercive subtyping mechanism, the undergeneration problems can be straightforwardly solved while still retaining the ability to rule out semantically infelicitous cases like *the ham sandwich walks*. In effect, many-sortedness in MTTs turns out to be superior than single sortedness in simple

[8] It is worth mentioning that subsumptive subtyping, the traditional notion of subtyping that adopts the subsumption rule (if $A \leq B$, then every object of type A is also of type B), is inadequate for MTTs in the sense that it would destroy some important properties of MTTs (see, for example, §4 of [18] for details).

type theory (at least in this respect). Furthermore, many inferences concerning the monotonicity on the first argument of generalized quantifiers can be directly captured using the subtyping mechanism. In effect an inference of the sort exemplified in the example (12) below, can be captured given that $[\![man]\!] < [\![human]\!]$:

(9) Some man runs $\Rightarrow$ Some human runs

Thus, an $x : [\![man]\!]$ can be used as an $x : [\![human]\!]$, and as such the inference goes through for 'free' in a way.[9]

2.4 Σ-types, Π-types and Universes

Dependent Σ-types. One of the basic features of MTTs is the use of Dependent Types. A dependent type is a family of types that depend on some values.

The constructor/operator Σ is a generalization of the Cartesian product of two sets that allows the second set to depend on values of the first. For instance, if $[\![human]\!]$ is a type and $male : [\![human]\!] \to Prop$, then the Σ-type $\Sigma h : [\![human]\!] \,.\, male(h)$ is intuitively the type of humans who are male.

More formally, if A is a type and B is an A-indexed family of types, then $\Sigma(A, B)$, or sometimes written as $\Sigma x{:}A.B(x)$, is a type, consisting of pairs (a, b) such that a is of type A and b is of type $B(a)$. When $B(x)$ is a constant type (i.e., always the same type no matter what x is), the Σ-type degenerates into product type $A \times B$ of non-dependent pairs. Σ-types (and product types) are associated projection operations π_1 and π_2 so that $\pi_1(a, b) = a$ and $\pi_2(a, b) = b$, for every (a, b) of type $\Sigma(A, B)$ or $A \times B$.

The linguistic relevance of Σ-types can be directly appreciated once we understand that in its dependent case, Σ-types can be used to interpret linguistic phenomena of central importance, like adjectival modification (see above for interpretation of modified CNs) [27].[10] For example, *handsome man* is interpreted as Σ-type (10), the type of handsome men (or more precisely, of those men together with proofs that they are handsome):

(10) $\Sigma m : [\![man]\!] \,.\, [\![handsome]\!](m)$

where $[\![handsome]\!](m)$ is a family of propositions/types that depends on the man m.

The use of Σ-types for dealing with adjectival modification will be further explained later on, when our proposal as regards the different classes of adjectives is going to be discussed.

[9] These kinds of inferences can be straightforwardly proven in Coq by using a standard analysis for quantifier *some* plus the subtyping relation $[\![man]\!] < [\![human]\!]$. See [4] for more details on treating NLIs as valid theorems in Coq.

[10] Σ-types also provide tools to give proper semantic interpretations of the so-called "Donkey-sentences" [28].

Dependent Π-types The other basic constructor for dependent types is Π. Π-types can be seen as a generalization of the normal function space where the second type is a family of types that might be dependent on the values of the first. A Π-type degenerates to the function type $A \rightarrow B$ in the non-dependent case. In more detail, when A is a type and P is a predicate over A, $\Pi x{:}A.P(x)$ is the dependent function type that, in the embedded logic, stands for the universally quantified proposition $\forall x{:}A.P(x)$. For example, the following sentence (11) is interpreted as (12):

(11) Every man walks.

(12) $\Pi x : [\![man]\!] \,.\, [\![walk]\!](x)$

Π-types are very useful in formulating the typings for a number of linguistic categories like VP adverbs or quantifiers. The idea is that adverbs and quantifiers range over the universe of (the interpretations of) CNs and as such we need a way to represent this fact. In this case, Π-types can be used, universally quantifying over the universe CN. (13) the type for VP adverbs[11] while (14) is the type for quantifiers:

(13) $\Pi A : \mathrm{CN}.\ (A \rightarrow Prop) \rightarrow (A \rightarrow Prop)$

(14) $\Pi A : \mathrm{CN}.\ (A \rightarrow Prop) \rightarrow Prop$

Further explanations of the above types are given after we have introduced the concept of type universe below.

Type Universes. An advanced feature of MTTs, which will be shown to be very relevant in interpreting NL semantics in general as well as adjectival modification specifically, is that of universes. Informally, a universe is a collection of (the names of) types put into a type [21].[12] For example, one may want to collect all the names of the types that interpret common nouns into a universe $\mathrm{CN} : Type$. The idea is that for each type A that interprets a common noun, there is a name $\overline{A}$ in CN. For example,

$$\overline{[\![man]\!]} : \mathrm{CN} \quad \text{and} \quad T_{\mathrm{CN}}(\overline{[\![man]\!]}) = [\![man]\!] \,.$$

In practice, we do not distinguish a type in CN and its name by omitting the overlines and the operator T_{CN} by simply writing, for instance, $[\![man]\!] : \mathrm{CN}$. Thus, the universe includes the collection of the names that interpret common nouns. For example, in CN, we shall find the following types:

(15) $[\![man]\!]$, $[\![woman]\!]$, $[\![book]\!]$, ...

[11] This was proposed for the first time in [15].

[12] There is quite a long discussion on how these universes should be like. In particular, the debate is largely concentrated on whether a universe should be predicative or impredicative. A strongly impredicative universe U of all types (with $U : U$ and Π-types) is shown to be paradoxical [7] and as such logically inconsistent. The theory UTT we use here has only one impredicative universe *Prop* (representing the world of logical formulas) together with an infinitely many predicative universes which as such avoids Girard's paradox (see [12] for more details).

(16) $\Sigma m : [\![man]\!] \,.\, [\![handsome]\!](m)$

(17) $G_R + G_F$

where the Σ-type in (16 is the proposed interpretation of 'handsome man' and the disjoint union type in (17) is that of 'gun' (the disjoint union of real guns and fake guns – see the discussion in §4).

Having introduced the universe CN, it is now possible to explain (13) and (14). The type in (14) says that for all elements A of type CN, we get a function type $(A \to Prop) \to Prop$. The idea is that the element A is now the type used. To illustrate how this works let us imagine the case of quantifier *some* which has the typing in (14). The first argument we need, has to be of type CN. Thus *some human* is of type $([\![human]\!] \to Prop) \to Prop$ given that the A here is $[\![human]\!]$: CN (A becomes the type $[\![human]\!]$ in $([\![human]\!] \to Prop) \to Prop$). Then given a predicate like $walk : [\![human]\!] \to Prop$, we can apply *some human* to get $[\![some\ human]\!]([\![walk]\!]) : Prop$.[13]

3 Σ-type Analysis of Modified CNs

Not much work focusing on adjectives has been done in formal semantics based on modern type theories. Adjectives are mainly studied in a Σ-type analysis on modified common nouns [27, 17], but not in general.[14] From this section on, we shall study adjectives in the MTT-based semantics more systematically.

In [27], the use of Σ-types to interpret common nouns modified by adjectives is proposed[15] and, in [14, 17], it is pointed out that subtyping is essential for such an interpretation of CNs as types to be adequate and proposed that coercive subtyping provide such a framework where one can have $\Sigma x{:}N.Adj(x) < N$, where N interprets a CN and Adj an adjective that modifies the CN.[16]

The Σ-type treatment is quite straightforward. CNs are interpreted as types and adjectives as predicates. Given that one has many types, the type of a predicate that interprets an adjective can vary according to the adjective. Thus, for example, *black* will be of type $[\![object]\!] \to Prop$ while *married* of type

[13] The idea of universes has been proved useful in accounting for NL phenomena from an MTT perspective. For example, in [3], the authors introduce a universe of Linguistic Types, *LType*, to capture the flexibility associated with NL coordination and in this paper we are going to use the universe CN to deal with some cases of adjectival modification.

[14] There is the interesting work by Jespersen & Primiero [9] on using a constructive type theory to deal with different classes of adjectives. However, the system they have used seems quite far from being an MTT as we have considered; it lacks multiple types or a subtyping mechanism, and they have considered CNs as predicates (following pretty much all of the Montagovian literature) rather than types.

[15] In a more traditional logic, Gupta [8] has suggested a special form of formulae $(K, x)A$, called *restrictions*, for interpreting modified CNs. Linking formulae to types, we can easily see the close correspondence between $(K, x)A$ and $\Sigma x{:}K.A$.

[16] In the Coq proof assistant, the record types are (top-level) Σ-types and used in [15].

$[\![human]\!] \to Prop$.[17] Given subtyping, *black man* can still be interpreted as $\Sigma m\colon [\![man]\!] \,.\, [\![black]\!](m)$ because $[\![man]\!] < [\![object]\!]$ and hence, by contravariance, *black* of type $[\![object]\!] \to Prop$ can also be regarded as of type $[\![man]\!] \to Prop$.

Now, intersective adjectives are associated with two main types of inference. The first one is not specific to intersective adjectives but is rather shared with subsective adjectives as well. It involves the entailment shown below:

(18) Adj(N) $\Rightarrow$ N.

Thus, according to the above, a black man is a man, a married man is also a man, and so on. Given that one can always have that the first projection π_1 of Σ-types be a coercion (see §2.3) the following always holds:

(19) From $\Sigma(N, Adj)$, infer N.

So, this first type of inference is easily taken care of by subtyping.

The second inference associated with intersective adjectives has to do with the fact that intersective modification not only entails that a given x of $Adj(N)$ is a N (e.g. a black man is a man), but further entails that $Adj(x)$ is also the case (e.g. that a black man is something black). Now, what does this mean in terms of inference? It implies that, for example, in *black man*, *black* can not only be applied to men (objects of $[\![man]\!]$) but also to any object whose type is a supertype of $[\![man]\!]$. Furthermore, no interpretation arises for the types that have no subtyping relation with $[\![man]\!]$; for instance, if *beautiful* is interpreted of type $[\![wowan]\!] \to Prop$ and $[\![man]\!]$ is not a subtype of $[\![woman]\!]$, there is no interpretation of *beautiful man*. Also, it is straightforward to see that the non-exsistence of a subtyping relation prohibits one from unwanted inferences. The inferences below are illustrative of the phenomenon:[18]

(20) A black man $\Rightarrow$ a black human

(21) A black man $\not\Rightarrow$ a black woman

The analysis proposed captures this fact given that coercions propagate through the various type constructors as well, e.g. Σ and Π. As such, besides the relation $[\![man]\!] < [\![human]\!]$, the subtyping relation $\Sigma([\![man]\!], [\![black]\!]) < \Sigma([\![human]\!], [\![black]\!])$ also holds via coercion propagation. Thus, the Σ-type analysis provides us with all the correct inferences as regards intersective adjectives.

It is not difficult to notice that an approach like the one given above will not work for some of the subsective adjectives. This is because subsective adjectives do not give rise to entailments like (20). In this sense, one might very well

[17] The discussion on how one builds the type ontology is of great importance but it is something that cannot be discussed here.

[18] We are a bit informal here, as an anonymous reviewer has noticed, saying that we should also deal with the determiner *a* in these cases. The determiner *a* is interpreted as the existential quantifier whose type is the same as the other quantifiers, i.e. (14). The inference (20), for example, is just saying that if m is of type $[\![black\ man]\!]$, it is also of type $[\![black\ human]\!]$.

argue that the treatment proposed for intersective adjectives will overgenerate for subsective adjectives and as such the Σ-type analysis must be abandoned in these cases. However, this is not the case and as we are going to explain, the Σ-type analysis can be maintained in the case of subsective adjectives as well. Let us see how.

The reason why subsective adjectives do not give rise to inferences like (20) has to do with the fact that they are only relevant for a particular class of words (CNs) they modify. Thus, a skilful surgeon is only skilful as a surgeon and not as a man or a human being. Implementing this idea, subsective adjectives like *large* can be given the type below:

(22) $\Pi A : \textsc{cn}.\ (A \rightarrow Prop)$

Using the above type, we have many instances of *large* depending on the choice of A. $large([\![man]\!])$ is of type $[\![man]\!] \rightarrow Prop$, $large([\![animal]\!])$ is of type $[\![animal]\!] \rightarrow Prop$, and so on. In this respect, we get different 'larges' as such for different As. Using this, one can achieve the meaning of subsective adjectives, i.e. that if something is large, it is only large for its class denoted by the CN (a large elephant is thus only large as an elephant). This way of treating subsective adjectives will correctly account for the inferences associated with subsective adjectives. In particular, inferences like (18) are taken care of via the usual first projection coercion of the Σ-type, while inferences similar to (20) are avoided given that the adjective is only meaningful with respect to the specific class in each case.

However, we are not done yet. This is because, a type like the above, cannot be used for cases like *skilful*. The reason is that *skilful* cannot have such a general type. If we assume such a type, we will be able to get interpretations for *skilful rock* or *skilful car*, which does not seem correct. *Skilful* in this respect must apply to CNs of type $[\![human]\!]$ or subtypes of this latter type, e.g. $[\![doctor]\!], [\![violonist]\!] < [\![human]\!]$. This problem can be solved as follows: one can introduce a subuniverse of $\textsc{cn}$ containing the names of the types $[\![human]\!]$ and its subtypes only. Let us call this universe $\textsc{cn}_H$, which is a subtype of $\textsc{cn}$: $\textsc{cn}_H < \textsc{cn}$. Now, we can propose the following type for an adjective like skilful:

(23) $\Pi A : \textsc{cn}_H.\ (A \rightarrow Prop)$

Similar cases can be treated accordingly.

4 Privative Adjectives

Besides intersective and subsective adjectives, there is another adjectival class that does not give rise to any of the inferences associated with the aforementioned classes. This class of adjectives, is further subdivided into privative and non-committal adjectives. The former give rise to inferences like (24) while the latter do not give rise to any inference whatsoever:

(24) Adj(N) $\Rightarrow \neg$ N.

The standard way of dealing with privative adjectives as well as with the other classes of adjectives within the Montagovian tradition is via meaning postulates (see [24] for example). According to these types of approaches, the inferences are captured by postulating that certain types of adjectives are associated with the specific inferences. In the case of privative adjectives ADJ of type $(e \to t) \to (e \to t)$, the meaning postulate would be:

(25) $\forall Q : e \to t \forall x : e.\ ADJ(Q, x) \supset \neg Q(x)$

It is worth mentioning that meaning postulates are needed for all adjectival categories within a Montagovian setting, with an exception when one assumes that intersective adjectives be of a lower $e \to t$ type but again this has the disadvantage of disrupting type uniformity [24]. Partee in the same paper, and using data from Polish NP-split phenomena goes on to argue that the class of privative adjectives does not really exist. The reasoning in [24] as well as in [25] is that the interpretation of privative adjectives is in fact subsective. Partee argues that in cases of privative modification the interpretation of the CN is coerced to include the denotations of CNs modified by privative adjectives. For example in the case of (26) and (27), Partee argues that the denotation of *fur* is expanded to include both *real* and *fake* furs:

(26) I don't care whether that fur is fake fur or real fur.

(27) I don't care whether that fur is fake or real.

The idea is that in the case of *fake fur*, *fur* is coerced to include fake furs as well, while in the second case it is not. The idea in itself is very intriguing and indeed plausible given the data.

What we are going to propose is to use the disjoint union type in MTTs to formalise the semantics of privative adjectives. This was first proposed in an unpublished note by the second author [19], which can arguably be regarded as formalising the above idea of Partee in an MTT. Let us see how this can be done by discussing the case of fake and real guns.

We first assume that G_R and G_F be the type of (real) guns and that of fake guns, respectively. Then,

$$G = G_R + G_F$$

is the type of all guns. It consists of the objects of the form $inl(r)$ and $inr(f)$, where $r : G_R$ and $f : G_F$. Furthermore, we declare the associated injection operators $inl : G_F \to G$ and $inr : G_R \to G$ as coercions:

$$G_R <_{inl} G \quad and \quad G_F <_{inr} G.$$

We contend that the above employment of disjoint union type, together with the above declaration of subtyping relations, gives an adequate semantics of the privative adjective *fake*.

For instance, we can now define the following predicates $real\_gun$ and $fake\_gun$ of type $G \to Prop$:

$$real\_gun(inl(r)) = True \quad and \quad real\_gun(inr(f)) = False;$$

$$fake\_gun(inl(r)) = False \quad and \quad fake\_gun(inr(f)) = True.$$

If is easy to see that, for any $g : G$,

(28) $real\_gun(g)$ iff $\neg fake\_gun(g)$.

Now, the following interpretations can be given (both are true): for $g : G_R$:

(29) $[\![g\ is\ a\ real\ gun]\!] = real\_gun(g)$

and for $f : G_F$,

(30) $[\![f\ is\ not\ a\ real\ gun]\!] = \neg real\_gun(f)$

Note that in the above, $real\_gun(f)$ is only well-typed because $G_F <_{inr} G$ and in fact we have $real\_gun(f) = real\_gun(inr(f)) = True$. Similarly, with the above, it is not difficult to see that the sentences like those below can easily be interpreted as expected:

(31) Is that gun real or fake?

(32) A fake gun is not a gun.

In the above, we have only considered guns but not other objects. One may have the desire to type the word real and fake directly so that they can be applied to other objects different from guns. A possibility is to consider a type *Object* (of all objects) of which, for example, G is a subtype:

$$G <_{gun} Object.$$

Employing *Object*, we could have:

(33) $real,\ fake : Object \rightarrow Prop$

and it is then easy to see that

$$G_R <_{gun\ \circ\ inl} Object \quad and \quad G_F <_{gun\ \circ\ inr} Object.$$

This allows us to give more general types (33) to *real* and *fake* so that we can cover cases like *fake car*, *real president* etc.

Please note that the above is also a rather welcomed result in that it predicts that a fake gun is an object (and not a fake object). It seems in this respect that the above MTT analysis of privative adjectives can produce further welcoming results due to the nature of the subtyping mechanism. Other privative adjectives like *imaginary* can be treated accordingly.

5 Non-committal Adjectives

Privative adjectives, as already mentioned, comprise one of the subcategories of non-subsective adjectives, the other being the class of non-committal adjectives as these are usually called within the Montagovian tradition. In this category, we find modal adjectives like *alleged*, *possible* and *potential.* According to Partee [24], these are the only adjectives that do not give rise to any inferences at all:[19]

(34) Adj(N) $\Rightarrow$?.

Adjectives like *alleged* (and similar ones like *potential* and *possible*) involve a flavour of modality missing from the other classes of adjectives. Ranta [27] discusses the use of the notion of context in MTTs in order to deal with phenomena that have traditionally been dealt with using possible worlds in the model-theoretic tradition. Ranta in discussing the various issues associated with epistemic logic, proposes the notion of belief contexts: a belief context is a sequence of assumptions that an agent p has made. More precisely, the belief context of an agent p, notation Γ_p, is a context of the form:

(35) $\Gamma_p = x_1 : A_1, ..., x_n : A_n$

Based on this, Ranta proposes the belief operator B_p, defined as

$$B_p A = \Pi\Gamma_p.\ A = \Pi x_1{:}A_1 ... \Pi x_n{:}A_n.\ A.$$

As a consequence, $B_p A$ is true if and only if A is true in Γ_p.

Now, an adjective like alleged can be interpreted as follows. Let $A_N : \text{CN}$ be the interpretation of a common noun N. Then, we interpret

(36) $[\![alleged\ N]\!] = \Sigma p{:}Human.\ B(p, A_N)$

where $B(p, A) = \Pi\Gamma_p.\ A$ with Γ_p being the belief context of $p : Human$.[20] Intuitively, the above says that, for some human being p, p believes that A_N (the semantics of N) is true.[21] For example, the following sentence (37) is interpreted as (38):

(37) John is an alleged criminal.

(38) $[\![John]\!] : [\![alleged\ criminal]\!] = \Sigma p : Human.\ B(p, [\![criminal]\!])$

Similar cases of adjectives seem in principle to be accountable within the same line of approach. On a more general note, the constructive notion of context that has been claimed by Ranta [26, 27] to be the equivalent of the notion of a possible world in model-theoretic semantics is an idea that we believe needs to be taken

[19] This of course does not mean that they are devoid of meaning. This is a separate issue.

[20] Note that, strictly speaking, p in B_p is a meta-level entity; we are abusing the notation here. Formally, we can use a universe U that contains the Π-types and inductively define $B : Human \to U \to U$. Details are omitted.

[21] This is the analog of a formula that involves existential quantifications. One may turn such types into propositions by means of the following operation: for any type A, $Exists(A) = \exists x{:}A.True$. Then, with this mechanism, (36) can be represented as the proposition $\exists p{:}Human.\ Exists(B(p, A_N))$.

into consideration more seriously. Such an approach may potentially provide us with a general account of intensionality. Indeed, a number of proposals have been put forth both by Ranta himself as well as other researchers building on work by Ranta.[22] We hope that our work will contribute towards this direction as well.

6 The case of *former*

In the last section, we shall deal with some temporal adjectives such as *former* and *past*. If we follow Partee [24] and assume that *former* behaves similarly to adjectives like *fake* or *imaginary*, then one is committed to a similar analysis for *former* as we have done in §4. Indeed one could propose an analysis for *former* within the same lines as the one proposed for *fake*, assuming the necessary modifications are made.

Another way to deal with *former* is not via the disjoint union type but rather via using an explicit $Time$ argument. Such an argument is independently needed if one wants to deal with any kind of tense or aspectual phenomenon. Whether this $Time$ parameter will be an argument of the verb or a parameter in a more complex argument, like for example an event argument is something that we will not discuss here. For the moment, let's assume a simple model of tense – a type $Time$ with an ordering relation $<$ (see, for example, [27]). Then we assume that some CNs are indexed by the time parameter. For example, instead just having a CN president, we have a family of types

$$president(t) : \text{CN},$$

indexed by $t : Time$. We further assume that $now : Time$ stand for the 'current time'; for example, $president(now)$ is the CN president at the current time.

With the above mechanisms available, we can now interpret CNs modified by former as follows: for example,[23]

(39) $[\![former\ president]\!] = \neg president(now) \wedge \exists t : Time.\ t < now \wedge president(t).$

In general, we have $[\![former]\!] : (Time \to \text{CN}) \to \text{CN}$, obtained by abstracting *president* in the above definition:[24] for any $p : Time \to \text{CN}$,

(40) $[\![former]\!](p) = \neg p(now) \wedge \exists t : Time.\ t < now \wedge p(t).$

[22] See for example the work by [9] on adjectives like alleged or the work by [2] on NL phenomena involving beliefs.

[23] For understandability of the readers who are unfamiliar with MTTs, we abuse the notation here, using $\neg A$ to stand for $A \to \emptyset$, $\wedge$ for $\times$ and $\exists$ for Σ. One may ignore these formal details.

[24] Note that this type does not give rise to *Prop* after functional application but rather to CN. This is compatible with the fact that this type of adjectives cannot appear in predicative positions. In case one thinks that this is not a semantic issue but rather a syntactic one, one can use a slightly different definition so that $[\![former]\!]$ has type $(Time \to \text{CN}) \to Prop$, preserving a kind of type uniformity across all adjectival classes.

With $president : Time \to \textsc{cn}$, we have $[\![former\ president]\!] = [\![former]\!]([\![president]\!])$.

The above use of dependent types in semantic interpretations may have the potential to be generalised. Further research is needed in this direction.

7 Conclusion

In this paper, we proposed an account of the various classes of adjectives within an MTT setting. We have shown that the Σ-type analysis for adjectives can cover the subsective and intersective classes adequately thanks to the use of the subtyping mechanism as well as the use of the notion of a universe. However, it was shown that privative adjectives require a different treatment and proposed to treat this type of adjectives via disjoint union types. This type of approach gives us the correct results as regards the inferences associated with these types of adjectives. Lastly, non-committal adjectives were discussed and an account that makes use of the constructive notion of context as approximating possible worlds was given.

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