

Three learnable models for the description of language

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Learnable

THREE MODELS FOR THE DESCRIPTION OF LANGUAGE*



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Outline

Methodological part

- Efficient learnability should be central to language theory
- Design representations to be intrinsically learnable
- Primitives based on objective features of the language

Technical part

Three specific models:

- Regular based on the residual languages
- Context-free based on the syntactic monoid
- Non context-free representations based on the Syntactic Concept Lattice

Outline

- 1 Learnability
- 2 Regular languages
- 3 Congruence class approaches
- 4 Lattice based approaches
- 5 Conclusion

Language and Automata Theory and Applications

Two objects

Languages

Subsets of Σ^*

Distributions, $A^* \times B^*$, trees, ...

Class of languages \mathcal{L}

Finite Representations

Grammars, Automata, semi-Thue systems ...

Class of representations \mathcal{G}

What is the proper relation between these?

Typical definition

Normal direction

Define a representation G

Function from representation to language: $G \rightarrow \mathcal{L}$

Non-terminal \rightarrow set of strings derived from non-terminal

Context free grammar $G \rightarrow$ context free language $L(G)$

Fundamental computation

is $w \in L(G)$?

We want this to be efficient

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Where do these grammars come from?

Applications

Which of the two objects do we know about?

- Natural languages, DNA sequences . . .
- We have information about the language.
- But we typically have no direct information about the representation.

The representation may be a man-made object that we use to describe the language

(With one exception – programming and mark-up languages.)

Unsupervised Learning

Fundamental problem of linguistics

Language theory has its roots in linguistics:

Chomsky's questions (1986)

- 1 What constitutes knowledge of a language?
- 2 How is this knowledge acquired by its speakers?

Unsupervised Learning

Fundamental problem of linguistics

Language theory has its roots in linguistics:

Chomsky's questions (1986)

- 1 What constitutes knowledge of a language?
 - 2 How is this knowledge acquired by its speakers?
- The fundamental problem of linguistics it to account for language acquisition
 - We have no direct information about the representations that are used.
 - The perceived hardness of this learning problem is “the existence proof for cognitive science” – Jerry Fodor.

PSGs were meant to be learnable

Chomsky (1968/2006)

“The concept of "phrase structure grammar" was explicitly designed to express the richest system that could reasonable be expected to result from the application of Harris-type procedures to a corpus.”

Go backwards

Put learnability first!

Opposite Direction

Given a language and we want to define a representation
Function from language to representation

$$L \rightarrow G(L)$$

From set of strings \rightarrow representational primitive of formalism

Ideally $L(G(L)) = L$.

Fundamental computation

Construct representation $G(L)$ from L

We want this to be efficient

Two problems of grammar induction

First problem

Information theoretic problems

A general problem of learning:

- Absence of negative data (Gold, 1967)
- VC-dimension (Vapnik, 1998), covering numbers
- Sparsity, Noise etc.

We know how to attack these problems: MDL, NPB, MaxEnt

Not specific to grammatical inference

Two problems of grammar induction

Second problem

Computational problems

Complexity of finding a good hypothesis given this information

- Gold (1978), Kearns and Valiant (1989) ...
- Often based on embedding cryptographic problems in learning problems
- Specific to certain classes of representation

This is the crucial problem: given good information about the language, can we efficiently construct a representation?

Overview

	Inefficient	Efficient
Positive data and MQs	Gold (1967)	?
Stochastic data	Horning (1969) Angluin (1988) Chater and Vitanyi (2007)	?

These results suggest the presence of probabilistic data largely compensate for the absence of negative data. (Angluin, 1988)

Objective representations

Program

Given a language L

- 1 Define a collection of sets of strings as primitives
- 2 Define a derivation relation, based on algebraic properties of these sets.
- 3 Define a representation based on this derivation relation

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Step 1

Make a representational decision

Residual languages

$$u^{-1}L = \{w \mid uw \in L\}$$

right congruence classes

$$u \cong v \text{ iff } u^{-1}L = v^{-1}L$$

$$[u]_R = \{v \mid u \cong v\}$$

Primitives

Finite set of strings K ; $\lambda \in K$

$$Q = \{[u]_R \mid u \in K\}$$

$$q_0 = [\lambda]_R$$

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$$q_0 = [\lambda]_R$$

Let's call the elements of Q "states"

Example

$$L = \{(ab)^*\}$$

- $[\lambda]_R = L$
- $[a]_R = aL$
- $[b]_R = \{b, bb, bab, \dots\}$ ($u^{-1}L = \emptyset$)

Define the goal

Basic property

If $u \in L$ then $[u]_R \subseteq L$

Let $F \subseteq Q$ be $\{[u] \in Q \mid u \in L\}$

- A representation defines a function from $\Sigma^* \rightarrow \{0, 1\}$
 $f(w) = 1$ iff $w \in L$
- Define a function from $\Sigma^* \rightarrow Q$
We want $\delta(w) = q$ iff $w \in q$

Step 2

Recursive derivation

Algebraic property of the primitives

It is a right congruence:

$$u \cong v \Rightarrow uW \cong vW$$

$$[u]_R \circ W = [uW]_R$$

Derivation of δ

Left to right derivation:

- $\delta(\lambda) = [\lambda]_R = q_0$
- If $\delta(u) = q = [v]$ then $\delta(ua) = [va]_R$ if $[va]_R \in Q$

Language defined

If $\delta(w) = q$ and $q \in F$ then $w \in \hat{L}(K, L)$

DFA

A long route to a familiar destination

- This is just a deterministic automaton
- If $\delta(w) \in q$ then $w \in q$
- We will always undergenerate: $L \subseteq \hat{L}(K, L)$
- As K increases the language increases

Language class

If K is finite, then $\hat{L}(K, L)$ is regular

If L is regular and K is big enough then $\hat{L}(L, K) = L$
(Myhill-Nerode theorem)

Learning

Inference problems

How can we select K or Q ?

How can we tell whether $u \cong v$?

Three solutions

- 1 restrict the class of languages
 $uw, vw \in L \Rightarrow u \cong v$ (Angluin, 1982)
- 2 pick a set of strings F
 $u^{-1}L \cap F = v^{-1}L \cap F$
(Angluin, 1987)
- 3 Probabilistic measure of the similarity between $u^{-1}L$ and $v^{-1}L$, L_∞ norm
(Ron et al. 1994), Clark and Thollard, 2004)

Summary of regular learning

Derived DFA starting from a representational assumption.

Minimal DFAs are learnable because there is a bijection between the representational primitives and some objectively defined sets of strings of the language.

Define a set of primitives	Right congruence classes
Derivation relation	Transition function
Language class	Regular languages
Inference algorithms	Testing right congruence

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Distribution

Classic idea from structuralist linguistics:

Context (or *environment*)

A context is just a pair of strings $(l, r) \in \Sigma^* \times \Sigma^*$.

$$(l, r) \odot u = lur$$

$$f = (l, r).$$

Special context (λ, λ)

Given a language $L \subseteq \Sigma^*$.

Distribution of a string

$$C_L(u) = \{(l, r) \mid lur \in L\} = \{f \mid f \odot u \in L\}$$

“Distributional Learning” models/exploits the distribution of strings;

Classic distributional learning

Congruence classes

$u \equiv v$ iff $C_L(u) = C_L(v)$

Write $[u]$ for class of u

Syntactic monoid

Σ^* / \equiv_L

$[u] \circ [v] = [uv]$

Example $(ab)^*$

$[\lambda], [bb], [a], [b], [ab], [ba]$

Finitely many congruence classes iff L is regular.





Observation table

K a set of strings and F a set of contexts

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
k_8										
k_7										
k_6										
k_5										
k_4										
k_3										
k_2										
k_1										

Observation table

K a set of strings and F a set of contexts

	(λ, λ)	(a, λ)	(λ, b)
λ			
a			
b			
ab			

Observation table

K a set of strings and F a set of contexts

$(aaabb, bccc)$ $(\lambda, abbccc)$ (aa, bbc) (abb, cc)
 (λ, λ) $(aaabbc, \lambda)$ $(aaab, bccc)$ $(aa, bbbc)$ $(abbb, cc)$

<i>bcc</i>								■
<i>aab</i>						■		
<i>bbcc</i>	■						■	
<i>bc</i>	■						■	
<i>abc</i>	■							
<i>aabb</i>	■				■			
<i>ab</i>	■				■			
<i>c</i>	■		■					■
<i>b</i>				■				
<i>a</i>	■			■		■		
λ	■	■				■		■

Observation table

K a set of strings and F a set of contexts

(λ, λ) $(aaabb, bccc)$ $(\lambda, abbccc)$ $(aa, bbcc)$
 (aa, bbc) (abb, cc) $(aaabb, \lambda)$ $(aaab, bccc)$ $(abbb, cc)$

<i>bcc</i>								■
<i>aab</i>							■	
<i>abc</i>		■						
<i>aabb</i>	■	■						
<i>ab</i>	■	■						
λ	■	■	■	■				
<i>bbcc</i>		■	■					
<i>bc</i>		■	■					
<i>c</i>		■			■			■
<i>b</i>						■		
<i>a</i>		■				■		■

Substitutable

$$L = \{a^n cb^n \mid n \geq 0\}$$

	(λ, λ)	(a, b)	(λ, cb)	(ac, b)	(a, λ)	(λ, b)	(aac, b)
$aacb$						■	
ac						■	
$acbb$				■			
cb				■			
$aacbb$	■	■					
acb	■	■					
c	■	■					
b						■	■
a			■				

Context free grammar

Suppose we have a grammar with non-terminals N, P, Q

- We have a rule $N \rightarrow PQ$
- This means that $Y(N) \supseteq Y(P)Y(Q)$.

Context free grammar

Suppose we have a grammar with non-terminals N, P, Q

- We have a rule $N \rightarrow PQ$
- This means that $Y(N) \supseteq Y(P)Y(Q)$.

Backwards

Given a collection of sets of strings X, Y, Z

Suppose $X \supseteq YZ$

Then we add a rule $X \rightarrow YZ$.

Congruence classes

Partition of the strings

Congruence classes have nice properties!

$$[u][v] \subseteq [uv]$$

$$[uv] \rightarrow [u][v]$$

$$[u] \circ [v] \rightarrow [u][v]$$

$$L = \{a^n b^n \mid n \geq 0\}$$

$$[a] = \{a\}$$

$$[abb] = \{abb, aabbb, \dots\}$$

$$[a][abb] = \{aabb, aaabbb, \dots\} \subseteq [aabb] = [ab]$$

So we have a rule $[ab] \rightarrow [a][aab]$

Example

Artificial

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
k_8										■
k_7										■
k_6										■
k_5				■	■	■				
k_4				■	■	■	■	■	■	
k_3						■	■	■		
k_2	■	■	■							
k_1	■									

Example

Artificial

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k_2	■	■	■							
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Example

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Example

$$L = \{a^n b^n \mid n \geq 0\}$$

Grammar

- $S \rightarrow [ab], S \rightarrow [\lambda]$

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Grammar

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- $[a] \rightarrow a, [b] \rightarrow b, [\lambda] \rightarrow \lambda$

Example

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Grammar

- $S \rightarrow [ab], S \rightarrow [\lambda]$
- $[a] \rightarrow a, [b] \rightarrow b, [\lambda] \rightarrow \lambda$
- $[ab] \rightarrow [aab][b], [ab] \rightarrow [a][b], [ab] \rightarrow [a][abb]$
- $[aab] \rightarrow [a][ab], [abb] \rightarrow [ab][b]$

Example

$$L = \{a^n b^n \mid n \geq 0\}$$

Grammar

- $S \rightarrow [ab], S \rightarrow [\lambda]$
- $[a] \rightarrow a, [b] \rightarrow b, [\lambda] \rightarrow \lambda$
- $[ab] \rightarrow [aab][b], [ab] \rightarrow [a][b], [ab] \rightarrow [a][abb]$
- $[aab] \rightarrow [a][ab], [abb] \rightarrow [ab][b]$
- Plus $[ba] \rightarrow [b][ba] \dots$
- Plus $[a] \rightarrow [\lambda][a] \dots$

Problem

Hard to tell whether $u \equiv_L v$

If we can figure out the congruence classes, we can just write down a grammar.

Congruence class results

Positive data alone

$lur \in L$ and $lvr \in L$ implies $u \equiv_L v$

Polynomial result from positive data. (Clark and Eyraud, 2005)

k - l substitutable languages, (Yoshinaka 2008)

Stochastic data

If data is generated from a PCFG

PAC-learn unambiguous NTS languages, Clark (2006)

Membership queries

An efficient query-learning result Clark (under submission)

Pick a finite set of contexts F

Test if $C_L(u) \cap F = C_L(v) \cap F$

Old concept

John Myhill, 1950 commenting on Bar-Hillel

I shall call a system *regular* if the following holds for all expressions μ, ν and all wffs ϕ, ψ each of which contains an occurrence of ν : If the result of writing μ for some occurrence of ν in ϕ is a wff, so is the result of writing μ for any occurrence of ν in ψ . Nearly all formal systems so far constructed are regular; ordinary word-languages are conspicuously not so.

Clark and Eyraud, 2005

A language is *substitutable* if $lur, lvr, l'ur' \in L$ means that $l'vr' \in L$.

Substitutable CF languages are polynomially learnable.

Why the delay?

Language class

Quite limited

Class of CNF CFGs where each non-terminal generates a congruence class. (multiple S symbols)

Includes

- All regular languages (syntactic monoid is finite)
- Dyck language
- $\{a^n b^n | n \geq 0\} \dots$

Many simple languages are not in this class:

- Palindromes over $\{a, b\}$
- $L = \{a^n b^n | n \geq 0\} \cup \{a^n b^{2n} | n \geq 0\}$
- $L = \{a^n b^n c^m | n, m \geq 0\} \cup \{a^m b^n c^n | n, m \geq 0\}$

Summary

Context free

Define a set of primitives

Derivation relation

Language class

Inference algorithms

Congruence classes

Syntactic monoid

Subclass of CF languages

Testing congruence

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Representational primitives

Strings

$$[u] = \{v \mid v \equiv_L u\}$$

The smallest possible sets

Partition

Contexts

$$I[l, r] = \{v \mid lvr \in L\}$$

These are the largest possible sets.

Overlap

Representational primitives

Strings

$$[u] = \{v \mid v \equiv_L u\}$$

The smallest possible sets

Partition

Contexts

$$I[l, r] = \{v \mid lvr \in L\}$$

These are the largest possible sets.

Overlap

Intersections

Given a set of contexts F

For every subset $C \subseteq F$

$$C' = \{w \mid \forall (l, r) \in C, lwr \in L\}$$

$$\bigcap_{(l,r) \in C} I[l, r]$$

Concepts

Maximal rectangles

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
k_8										■
k_7										■
k_6										■
k_5				■	■	■				
k_4				■	■	■	■	■	■	
k_3						■	■	■		
k_2	■	■	■							
k_1	■									

Concepts

Maximal rectangles

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k_2	■	■	■							
k_1	■									

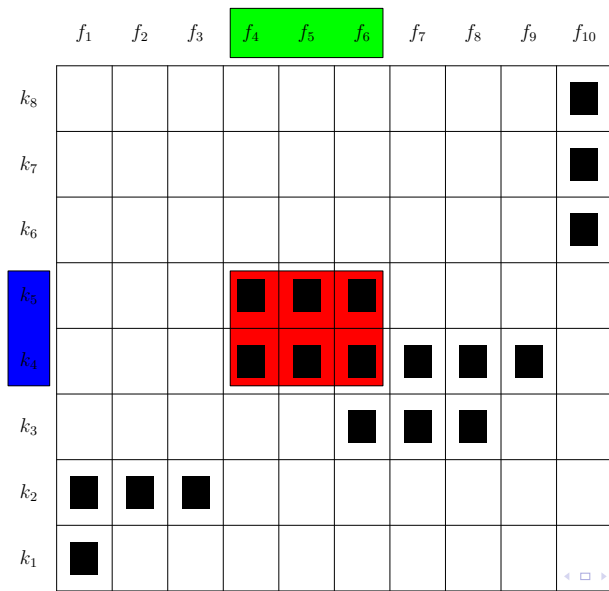
Concepts

Maximal rectangles

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k_1	■									

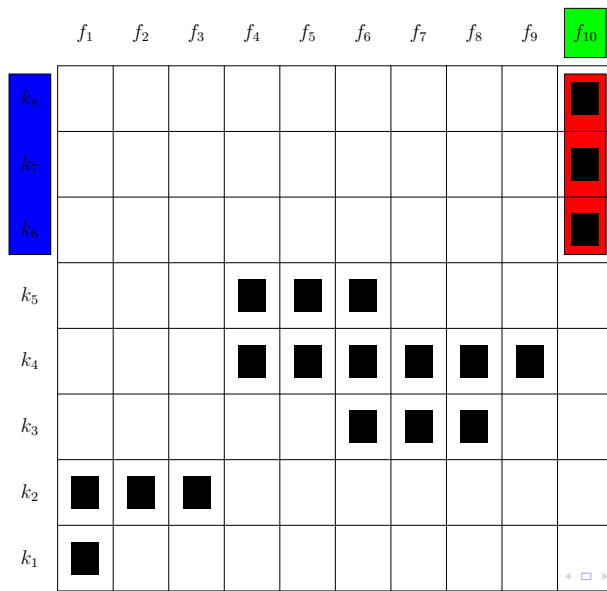
Concepts

Maximal rectangles



Concepts

Maximal rectangles



Concepts

Maximal rectangles

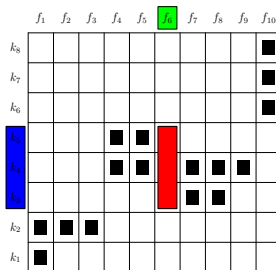
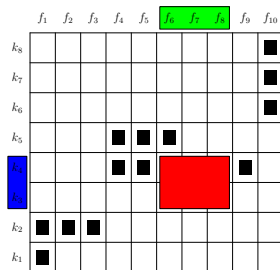
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Concepts

Maximal rectangles

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k_3						■	■	■		
k_2	■	■	■							
k_1	■									

Partial order



Top and bottom

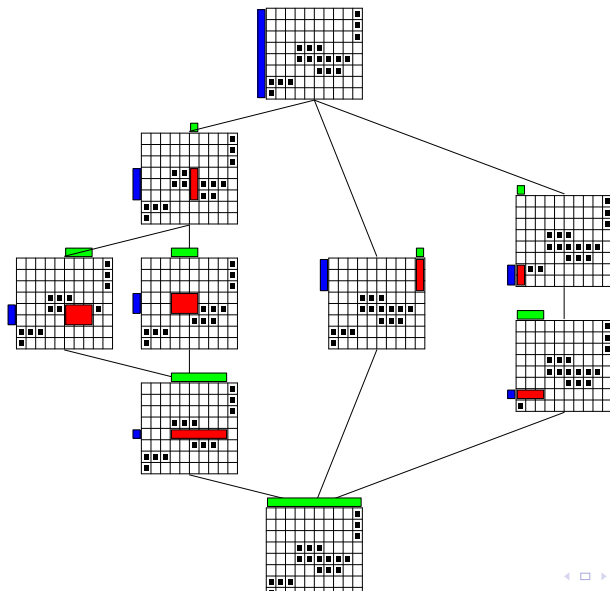
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k_6										■
k_5				■	■	■				
k_4				■	■	■	■	■	■	
k_3						■	■	■		
k_2	■	■	■							
k_1	■									

Top and bottom

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
k_8										■
k_7										■
k_6										■
k_5				■	■	■				
k_4				■	■	■	■	■	■	
k_3						■	■	■		
k_2	■	■	■							
k_1	■									

Complete Lattice

Formal Concept Analysis



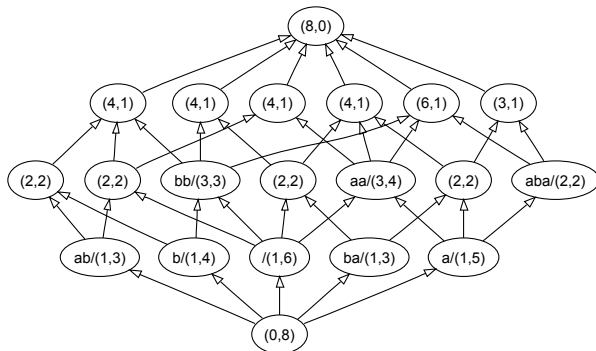
Lattice

Palindrome language over a, b

	(a, λ)	(aa, λ)	(ba, λ)	(λ, a)				
	(λ, λ)	(ab, λ)	(b, λ)	(λ, b)				
aba	■		■					
bb	■				■		■	
ba		■	■				■	
ab					■	■		■
aa	■	■		■				■
b	■				■	■	■	
a	■	■	■	■				■

Lattice

Many rectangles



Formally

Polar maps

$$S' = \{(l, r) \in F : \forall w \in S \ lwr \in L\}$$

$$C' = \{w \in K : \forall (l, r) \in C \ lwr \in L\}$$

Concept

Ordered pair $\langle S, C \rangle$

- $S \subseteq K$ the set of strings
- $C \subseteq F$ is a set of contexts

$$S' = C \text{ and } C' = S$$

$$\mathcal{C}(S) = \langle S'', S' \rangle$$

Relation to CFGs

Define

Given a CFG G for each non-terminal N

- Yield: $Y(N) = \{w \mid N \xRightarrow{*} w\}$
- Contexts: $C(N) = \{(l, r) \mid S \xRightarrow{*} lNr\}$.

Clearly $C(N) \odot Y(N) \subseteq L$

Each non-terminal will be a rectangle – but not necessarily maximal.

Technical detail

- These rectangles are “concepts” which form a complete lattice $\mathfrak{B}(K, L, F)$
- We use a concatenation operation $X \circ Y$ and a lower bound $X \wedge Y$.

Concatenation

$$\langle S_1, C_1 \rangle \circ \langle S_2, C_2 \rangle = \langle (S_1 S_2)'', (S_1 S_2)''' \rangle$$

Given two sets of strings S_x, S_y

Concatenate them $S_x S_y$

Find the shared set of contexts C_{xy}

Result is the concept defined by C'_{xy}

Dyck language

$\lambda, ab, abab, aabb, abaabb \dots$

	(λ, λ)	(a, λ)	(λ, b)
λ			
a			
b			
ab			

- $L = \langle \{\lambda, ab\}, (\lambda, \lambda) \rangle$
- $A = \langle \{a\}, (\lambda, b) \rangle$
- $B = \langle \{b\}, (a, \lambda) \rangle$
- \top
- \perp

Dyck language

$\lambda, ab, abab, aabb, abaabb \dots$

- $L = \langle \{\lambda, ab\}, (\lambda, \lambda) \rangle$
- $A = \langle \{a\}, (\lambda, b) \rangle$
- $B = \langle \{b\}, (a, \lambda) \rangle$
- \top
- \perp

	\top	L	A	B	\perp
\top	\top	\top	\top	\top	\perp
L	\top	L	A	B	\perp
A	\top	A	\top	L	\perp
B	\top	B	\top	\top	\perp
\perp	\perp	\perp	\perp	\perp	\perp

Goal

Predict which concept a string is in:

- Define function $\phi : \Sigma^* \rightarrow \mathfrak{B}(K, L, F)$
- A string w is in the language if $\phi(w)$ has the context (λ, λ) .
- We want $\phi(w) = \langle S, C \rangle$ to mean that $C_L(w) \cap F = C$.

Recursive definition

- $\phi(a) = C(a)$ (look it up)
- $\phi(ab) = \phi(a) \circ \phi(b)$
- $\phi(abc) = \phi(ab) \circ \phi(c)$, OR $\phi(a) \circ \phi(bc)$

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- $\phi(abc) = \phi(ab) \circ \phi(c) \wedge \phi(a) \circ \phi(bc)$

Distributional lattice grammars

Derivation: efficient $\mathcal{O}(|w|^3)$ algorithm

Definition

A distributional lattice grammar (DLG) is a tuple $\langle K, D, F \rangle$

- K is a finite subset of strings that includes Σ and λ
- F is a finite set of contexts that includes (λ, λ)
- D is a finite subset of $F \odot KK$

Definition

$\phi : \Sigma^* \rightarrow \mathfrak{B}(K, D, F)$.

- for all $a \in \Sigma$, $\phi(a) = \mathcal{C}(a)$
- for all w with $|w| > 1$,

$$\phi(w) = \bigwedge_{u,v \in \Sigma^+ : uv=w} \phi(u) \circ \phi(v)$$

Derivation example

Dyck language

$$\{\lambda, ab, aabb, abab, aaababbb \dots\}$$
$$F = \{(\lambda, \lambda), (\lambda, b), (a, \lambda)\}$$
$$K = \{\lambda, a, b, ab\}$$

- $\top = \langle K, \emptyset \rangle$
- $\perp = \langle \emptyset, F \rangle$
- $\mathbf{L} = \langle \{\lambda, ab\}, \{(\lambda, \lambda)\} \rangle$
- $\mathbf{A} = \langle \{a\}, \{(\lambda, b)\} \rangle$
- $\mathbf{B} = \langle \{b\}, \{(a, \lambda)\} \rangle$

Derivation example

Dyck language

$\{\lambda, ab, aabb, abab, aaababbb \dots\}$

$F = \{(\lambda, \lambda), (\lambda, b), (a, \lambda)\}$

$K = \{\lambda, a, b, ab\}$

- 1 $\phi(a) = A$
- 2 $\phi(ab) = \phi(a) \circ \phi(b) = \mathbf{L}$, $\phi(aa) = \top \dots$
- 3 $\phi(aab) = \phi(a) \circ \phi(ab) \wedge \phi(aa) \circ \phi(b) = A$
- 4 \dots
- 5 $\phi(aaababbb) = \phi(a) \circ \phi(aababbb) \wedge \dots = \mathbf{L}$

Example

$$L = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^m b^n c^n \mid n, m \geq 0\}$$

$$\begin{array}{cccccc} (aaabb, bccc) & (\lambda, abbccc) & (aa, bbc) & (abb, cc) & & \\ (\lambda, \lambda) & (aaabbc, \lambda) & (aaab, bccc) & (aa, bbbc) & (abbb, cc) & \end{array}$$

<i>bcc</i>									■
<i>aab</i>							■		
<i>bbcc</i>	■							■	
<i>bc</i>	■							■	
<i>abc</i>	■								
<i>aabb</i>	■					■			
<i>ab</i>	■					■			
<i>c</i>	■		■						■
<i>b</i>					■				
<i>a</i>	■			■			■		

Example

$$L = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^m b^n c^n \mid n, m \geq 0\}$$

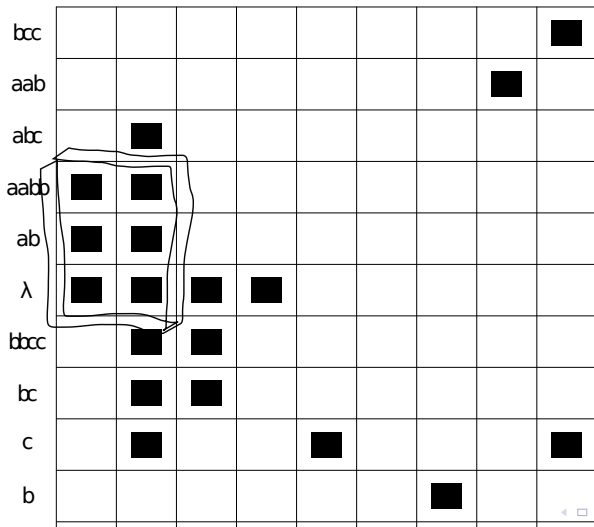
$$\begin{array}{cccc}
 (\lambda, \lambda) & (aaabb, bccc) & (\lambda, abbccc) & (aa, bbcc) \\
 (aa, bbc) & (abb, cc) & (aaabbc, \lambda) & (aaab, bccc) & (abbb, cc)
 \end{array}$$

<i>bcc</i>									■
<i>aab</i>								■	
<i>abc</i>		■							
<i>aaab</i>	■	■							
<i>ab</i>	■	■							
λ	■	■	■	■					
<i>bbcc</i>		■	■						
<i>bc</i>		■	■						
<i>c</i>		■			■				■
<i>b</i>							■		

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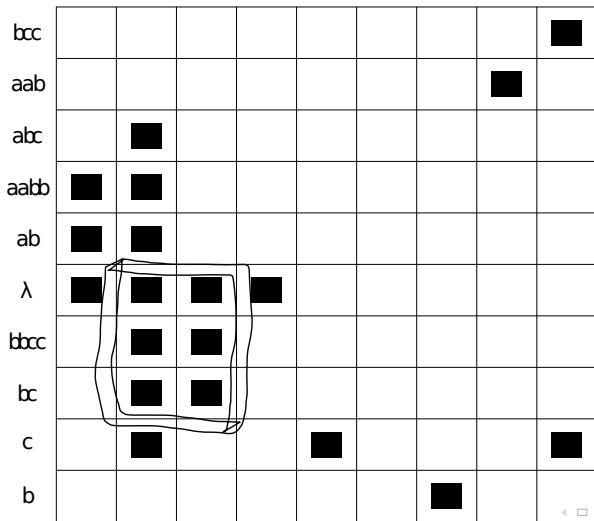
(λ, λ) $(aaabb, bccc)$ $(\lambda, abbccc)$ $(aa, bbcc)$
 $(aa, bbcc)$ (abb, cc) $(aaabbcc, \lambda)$ $(aaab, bccc)$ $(abbb, cc)$



Example

$$L = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^m b^n c^n \mid n, m \geq 0\}$$

(λ, λ) $(aaabb, bccc)$ $(\lambda, abbccc)$ $(aa, bbcc)$
 $(aa, bbcc)$ (abb, cc) $(aaabbcc, \lambda)$ $(aaab, bccc)$ $(abbb, cc)$



Learnability I

Search

Language is defined by choice of K and F
How can we find suitable K and F ?

Lemma 1

as we increase K the language defined by $\langle K, L, F \rangle$ decreases monotonically
It will always converge to a subset of L in a finite time

Lemma 2

As we increase the set of contexts F the language monotonically increases.
Any sufficiently large set of contexts will do.

Search problem is trivial

Naive Algorithm

Start with $F = \{(\lambda, \lambda)\}$, $K = \Sigma \cup \{\lambda\}$

- If we see a string that is not in our hypothesis, the hypothesis is too small, and we add contexts to F
- Add strings to K if it will change the lattice at all.

Clark, (CoNLL, 2010)

DLGs can be learnt from positive data and MQs

Polynomial update time

Power of Representation

Language class

Let \mathcal{L} be the set of all languages L such that there is a *finite* set of contexts F s.t. $L = L(\mathfrak{B}(\Sigma^*, L, F))$

Learnable class includes

- 1 All regular languages
- 2 Some but not all CFLs (all the examples so far)
- 3 Some non context free languages

Not in DLG?

$$L = \{a^n b \mid n > 0\} \cup \{a^n c^m \mid m > n > 0\}$$

	(a, λ)	(aa, λ)	(aaa, λ)	(a^4, λ)	(a^5, λ)	(a^6, λ)	(a^7, λ)
c^9	■	■	■	■	■	■	■
c^8	■	■	■	■	■	■	■
c^7	■	■	■	■	■	■	■
c^6	■	■	■	■	■	■	■
c^5	■	■	■	■	■	■	
c^4	■	■	■	■	■		
ccc	■	■	■	■			
cc	■	■	■				
c	■	■					
b	■	■	■	■	■	■	■

Context sensitive example

Let $M = \{(a, b, c)^*\}$, we consider the language
 $L = L_{abc} \cup L_{ab} \cup L_{ac}$ where $L_{ab} = \{wd \mid w \in M, |w|_a = |w|_b\}$,
 $L_{ac} = \{we \mid w \in M, |w|_a = |w|_c\}$,
 $L_{abc} = \{wf \mid w \in M, |w|_a = |w|_b = |w|_c\}$.

$F = \{(\lambda, \lambda), (\lambda, d), (\lambda, ad), (\lambda, bd), (\lambda, e), (\lambda, ae), (\lambda, ce), (\lambda, f), (ab, \lambda)\}$,

This is in the learnable class.

Syntactic concept lattice

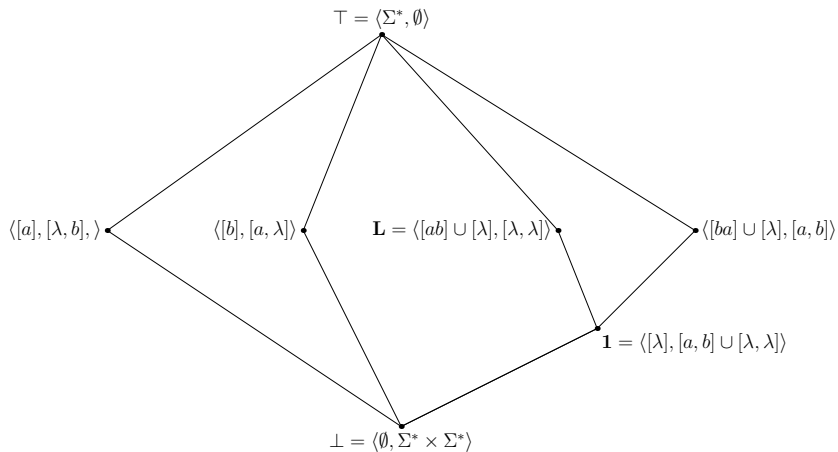
Infinite limit

Let $K \rightarrow \Sigma^*$ and $F \rightarrow \Sigma^* \times \Sigma^*$

$\mathfrak{B}(K, L, F) \rightarrow \mathfrak{B}(L)$

- $\mathfrak{B}(L)$ is the *syntactic concept lattice*
- Same construction as the Universal Automaton for regular languages
- $\mathfrak{B}(L)$ is finite iff L is regular

$$L = (ab)^*$$



Complete residuated lattice

- $X = \langle S_x, C_x \rangle$ and $Y = \langle S_y, C_y \rangle$ are concepts.
- Then define the residual $X/Y = \mathcal{C}(C_x \odot (\lambda, S_y))$
- $Y \setminus X = \mathcal{C}(C_x \odot (S_y, \lambda))$

These are unique, and satisfy the following conditions:

Lemma

$Y \leq X \setminus Z$ iff $X \circ Y \leq Z$ iff $X \leq Z/Y$.

Useful for displaced constituents in natural language: “the book that I told you to read”

Outline

- 1 Learnability
- 2 Regular languages
- 3 Congruence class approaches
- 4 Lattice based approaches
- 5 Conclusion**

General learning principle

Applies to other language classes:

- Finite languages – $\{w\}$
- k -locally testable languages
- Planar languages (Clark, Costa Florencio and Watkins, 2006)

The Future

Various relations

- $u \sim v$ iff $uv \in L$
- $(l, r) \sim u$ iff $lur \in L$

The Future

Various relations

- $u \sim v$ iff $uv \in L$
- $(l, r) \sim u$ iff $lur \in L$
- $(l, m, r) \sim (u, v)$ iff $lumvr \in L$ (Yoshinaka, 2009)
- Equational theory and relations to CG and pregroup grammars
- Decidability
- Better understanding of the class of languages

Learnability

- Grammatical inference is crucial and representation classes need to be designed to be learnable
- The structure of the representation should be based on the structure of the data
- Applying this approach gives efficient algorithms
 - Congruence based approaches using CFGs
 - Syntactic concept lattice
 - Distributional Lattice Grammars
- These are the only efficient algorithms for large classes of context free languages.