Embedding graphs containing K_5-subdivisions on the torus

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We are interested in an efficient computer algorithm to decide whether a graph G embeds on the torus. Classically, a torus embedding algorithm starts with a subgraph of G isomorphic to a subdivision of K_5 or $K_{3,3}$ and tries to extend one of its embeddings on the torus to an embedding of the whole graph G. Recently, we have shown a modification of this approach: non-planar graphs which do not contain a certain kind of $K_{3,3}$-subdivision are much easier to embed on the torus (see [2]). This provides an efficient means of handling the case of K_5 embeddings.

Developing the ideas, given a non-planar graph G containing a K_5-subdivision subgraph, we show that it is possible either to transform the K_5-subdivision into a certain type of $K_{3,3}$-subdivision, or else to reduce the toroidality testing problem for G to a small constant number of planarity checks and, eventually, rearrangements of planar embeddings. In [1], it is shown how to consider efficiently only one K_5-subdivision in the input graph G to decide whether G is embeddable on the torus. This makes it possible to detect a bigger class of toroidal and non-toroidal graphs than in [2].

References.

[1] A. Gagarin, W. Kocay, Embedding graphs containing K_5-subdivisions on the torus, *J. Combin. Math. Combin. Comput.* **80** (2012) 207-223.
[2] A. Gagarin, W. Kocay, Embedding graphs containing K_5-subdivisions, *Ars Combin.* **64** (2002) 33-49.

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