# **PCC 2009** 20th Postgraduate Combinatorial Conference





 $\mathbf{22}-\mathbf{24} \ \mathbf{June} \ \mathbf{2009}$ 

Royal Holloway, University of London

### Welcome to the Postgraduate Combinatorial Conference 2009 at the Royal Holloway, University of London!

We are very pleased to see so many of you here, and hope that everyone will have an interesting and enjoyable time at the conference. We are particularly delighted to welcome our invited speakers Prof. Andrew G. Thomason, Prof. Dave Cohen, Dr. Anders Yeo, Prof. Derek Smith and Dr. Angela Koller. All of us organisers will be happy to help you with any queries. Best wishes for the conference and beyond,

Arezou, EunJung and Sian.

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### **General Information**

### **Invited Speakers**

Prof. Andrew G. Thomason. Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, Cambridge, England

Prof. Derek Smith. Faculty of Advanced Technology, University of Glamorgan, Pontypridd, Wales.

Prof. Dave Cohen. Department of Computer Science, Royal Holloway University of London, England

Dr. Anders Yeo. Department of Computer Science, Royal Holloway University of London, England.

Dr. Angela Koller. Speaker from industry.

#### Sponsors

We are immensely grateful for the very generous support from our sponsors:

- The London Mathematical Society (LMS): www.lms.ac.uk
- The British Combinatorial Committee (BCC): www.maths.qmul.ac.uk/ ~pjc/bcc

Their financial contributions reduced the registration fees we have had to charge considerably.

#### Talks

All talks will be held in the Management Annexe lecture theatre MX01 (13 on the map, page 6). A computer, a data projector, an overhead projector, a whiteboard and pens will be provided.

#### Accommodation

Delegate accommodation is provided in Runnymede (43 on the map, page 6), check-in with Customer Services at the Hub, from Monday 22 July until the morning of Wednesday 24 July. Checkout before 10:00am on Wednesday 24 July.

#### Catering

- Breakfast will be served on Tuesday and Wednesday in The Athlone HUB (number 41 on the map, page 6) 07:30 08:59.
- Morning and afternoon tea & coffee breaks will take place in the Moore Annexe (13 on the map, page 6).
- Lunch on Monday and Tuesday will be served in the Moore Annexe 12:00 14:00.
- The evening meal on Monday and the Conference Dinner on Tuesday will be served in the SCR 18:00 to 20:00 (1 on the map, page 6).

### **Internet Access**

Connect to the wireless network **CampusNet** using the following details: First Name: **Combinatorial** Last Name: **Conference** Password: **RHUL** 

#### PCC 2010

This is the 20th Postgraduate Combinatorial Conference (PCC). In previous years the conference was held at the following universities:

- PCC 2008 at the University of Warwick,
- PCC 2007 at the University of St Andrews,
- PCC 2006 at the University of Glamorgan,
- PCC 2005 at the University of Oxford,
- PCC 2004 at Queen Mary, University of London,
- PCC 2003 at the University of Nottingham.

Why don't you organise next year's PCC? Feel free to speak to any of this year's organisers about what's involved.

6 1 General Information

### **Organising Committee**

Arezou Soleimanfallah , Royal Holloway, University of London EunJung Kim, Royal Holloway, University of London Sian Jones, University of Glamorgan

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# Programme

# Monday 22 June

$\begin{array}{l} 11:00 - 12:15\\ 12:15 - 12:45\\ 12:45 - 13:30 \end{array}$	Check-in with Customer Services at the Hub Meet at Moore Annex Lunch at the Moore Annex
Session 1 13.30 - 13.35 13.35 - 14.35 14.40 - 15.00	Chair: EunJung Kim Welcome by Dr Adrian Johnstone Dr. Anders Yeo: Transversals in Hypergraphs, Total Domi- nation in Graphs and Other Related Problems Arezou Soleimanfallah: A new kernel of size $2k - c$ for the Vertex Cover Problem
15.00 - 15.30	Tea & coffee
Session 2 15.30 - 15.50 15.50 - 16.10 16.10 - 16.30	Chair: Ryan Davies Linzy Phillips: Encoding Information in a Sudoku Grid Sian Jones: Unavoidable Sets in Sudoku and Quasi-Magic Sudoku Grids Niema Aboluion: Construction of DNA codes
Session 3 16.45 – 17.45	Chair: Emil Vaughan <b>Prof. Andrew G. Thomason</b> : 2-Coloured Graphs, Multi- graphs and Hereditary Properties
18.00 -	Dinner & drinks at the Founder's Building - SCR

# $\mathbf{2}$

### Tuesday 23 June

07.30 - 08.59	Breakfast at The Hub
Session 4 09.30 - 10.30 10.30 - 11.00	Chair: Matthew White <b>Prof. Dave Cohen</b> : Arrow's Theorem Showing That There is No Fair Voting System <i>Tea &amp; coffee</i>
Session 5 11.10 - 11.30 11.30 - 11.50 11.50 - 12.10	Chair: Manuela Heurer <b>Tony Forbes</b> : Eleven billion STS(19)s <b>Costas Psomas</b> : Biembeddings of Steiner triple systems us- ing the Bose construction <b>Matthew White</b> : 2-Factors in Hamiltonian Graphs
$\begin{array}{r} 12.15 - 12.45 \\ 12.45 - 13.30 \end{array}$	Photos Lunch at the Moore Annex
Session 6 13.50 – 14.50 15.00 – 16.30	Chair: Sian Jones <b>Prof. Derek Smith</b> : Old and New Problems in Coding The- ory <i>Tea &amp; coffee</i>
Session 7 16.45 – 17.05 17.05 – 17.25 17.25 – 17.45	<ul> <li>Chair: Linzy Phillips</li> <li>EunJung Kim: Algorithm for Finding k-Vertex Out-trees and its Application to k-Internal Out-branching Problem</li> <li>Daniel Karapetyan: A Memetic Algorithm for the Multidimensional Assignment Problem</li> <li>John Faben: Reducing Graphs by Involutions</li> </ul>
19.00 -	Conference dinner at the Founder's Building - SCR

10 2 Programme

### Wednesday 24 June

07.30 - 08.59Breakfast at The HubSession 8Chair: Arezou Soleimanfallah09.05 - 10.05Dr. Angela Koller: Graph Theory in Industry

10.05 - 11.00 Goodbye Tea & coffee

### Abstracts of Invited Talks Transversals in hypergraphs, total domination in graphs and other related problems

Anders Yeo Royal Holloway, University of London anders@cs.rhul.ac.uk

(joint work with Michael Henning, Arezou Soleimanfallah and Stephan Thomasse)

A set S of vertices in a graph G is a total dominating set of G if every vertex of G is adjacent to some vertex in S. The minimum cardinality of a total dominating set is called the *total domination number*.

A hypergraph, H, contains a vertex set denoted by V(H) and an edge set denoted by E(H). Every hyper-edge in H is a subset of the vertices. For graphs these sets all have size tw0, but for hypergraphs they can have any size. A transversal (also called a hitting set) in a hypergraph, H, is a set of vertices  $T \subseteq V(H)$ , such that every hyper-edge in E(H) contains at least one vertex from T.

We will both give bounds on the size of transversals in several kind of hypergraphs and show how these bounds can be used to obtain many different kind of bounds for the total domination number of a graph with properties such as (i) minimum degree 3 or 4, (ii) 2-connected, (iii) minimum degree 2, containing no induced 6-cycles and (iv) minimum degree 3, containing no 4-cycle.

The area of *fixed parameter tractable algorithms* is new, interesting and growing rapidly. As finding transversals in 3-uniform hypergraphs (i.e. all edges contain 3 vertices) has many application, we will mention a fixed parameter tractable algorithm for this problem. This algorithm can immediately be used in areas such as computational biology (related to phylogenetic trees) and tournaments (finding a minimum feedback vertex set). The time complexity of our algorithm beats all previously know algorithms.

We finally mention several open problems and conjectures.

Keywords: Hitting Set, Transversal, Hypergraph, Total Domination.

### 2-coloured graphs, multigraphs and hereditary properties

Andrew Thomason University of Cambridge a.g.thomason@dpmms.cam.ac.uk

(joint work with Ed Marchant)

Several different lines of study, such as those of Ramsey games, the probability of hereditary graph properties, and the computational complexity of graph testing, all lead to the following extremal problem for 2-coloured graphs.

A 2-coloured graph is a graph whose edges are coloured red, blue or green. A 2-coloured graph H is contained in another one G if H is a subgraph of Gin a way that respects the colours, except that green is a wildcard, meaning that a green edge of G can represent an edge of H of any colour. The weight of the graph G is the sum of its edge weights, these being p, q or 1 = p + qaccording as the edge is red, blue or green. The extremal problem is to find the maximum weight of a graph G that does not contain some forbidden graph H.

It is possible to prove quite a lot about the broad structure of the extremal configurations, which makes it possible to find the maximum weight for many graphs H, so yielding progress in the applications mentioned. But many questions remain unresolved.

MSC2000: 05C35.

Keywords: hereditary graph properties, Szemerédi's regularity lemma, Ramsey games.

### Arrow's Theorem Showing That There is No Fair Voting System

### Dave Cohen Department of Computer Science, Royal Holloway University of London d.cohen@rhul.ac.uk

Back in 1953 an economist, Ken Arrow, proved a key result concerning social welfare. The problem he addressed was how individual preferences can be aggregated to derive social preference.

Of course this is a problem in combinatorics. The existence of a function from a collection of individual quasi-orderings to a single quasi-ordering which satisfiescertain properties. Arrow proved that, under very natural conditions, such a function does not exist.

This has had implications on social and economic theory ever since and many economists have come back to the problem. The original proof, which was notparticularly illuminating, has been considerably simplified since Arrow's time and many systems for avoiding the impossibility have been proposed. Papers on the subject are still being published in 2009.

In this talk we will consider such social welfare functions. We will give twoproofs of the impossibility result: one simple and direct, the other usinginteresting combinatorics and appealing to well-known results. We will also explain a subsequent issue - the liberal paradox - which has tested thinkers both on the left of socio-economic thinking.

### Old and new problems in coding theory

Derek H. Smith Division of Mathematics and Statistics, University of Glamorgan, Pontypridd, Mid Glamorgan, CF37 1DL dhsmith@glam.ac.uk

This talk describes some problems in coding theory arising from areas as diverse as data compression, genetics and interference mitigation in codedivision multiple-access (CDMA) radio systems.

Huffman codes are variable length codes designed for data compression. Although better data compression techniques are available, they still find a role in combination with these techniques. In common with most data compression methods they can potentially suffer from massive error propagation. Alternative codes such as HE-codes and T-codes have been proposed, but do not always exist. Codes known as OT-codes exist for all required length vectors and resynchronize more quickly. It is suggested that OT-codes should always be used in preference to Huffman codes. An important problem that remains concerning OT-codes is outlined.

Coding theory is beginning to find significant application in Biology. Some of these applications are outlined, in particular to the design of synthetic DNA strands.

In synchronous or quasi-synchronous CDMA systems the rows of Hadamard matrices form codewords or are used in the construction of codewords. The fact that the inner product of rows is zero is exploited to avoid interference. If more codewords are required several Hadamard matrices can be used, but the inner product of rows will no longer be zero and interference will be greater. It is shown that it is possible to select four Hadamard matrices and assign the rows to a tessellation of hexagonal cells in such a way that rows assigned to the same cell or to adjacent cells have zero inner product. The extension of the idea to asynchronous CDMA is discussed.

#### MSC2000: 94A29, 92D20, 05B20.

Keywords: Variable length codes, DNA codes, CDMA, Hadamard matrices.

### Angela Koller JLT Reinsurance Brokers Limited angela.koller@jltre.com

In this session we will hear how a PhD in Graph Theory can be useful in the Insurance Industry. Angela will briefly talk about her background, how she found her current job in the city and what areas in General Insurance she is working on. She will then focus on a few particularly interesting problems, where using optimisation, general mathematical knowledge and transferrable skills all played an important role.

Keywords: General Insurance, Industry, Optimisation, Geometry.

### Abstracts of Contributed Talks

Session 2

# A new kernel of size 2k - c for the Vertex Cover Problem

Arezou Soleimanfallah Royal Holloway University of London arezou@cs.rhul.ac.uk

(joint work with Anders Yeo)

Given a graph G = (V, E), a vertex cover is a set  $VC \subset V(G)$  such that for every edge  $uv \in E(G)$ , we have  $\{u, v\} \cap VC \neq \emptyset$ . The Parameterised Vertex Cover is defined as: given a graph G = (V, E) and a positive integer k, does there exist a vertex cover VC in G such that  $|VC| \leq k$ ? This problem belongs to the class of FPT problems. Vertex Cover is amongst the wellstudied problems of graph theory. Amongst the motivations for studying this problem lies its application in real life. For example in computational biology, the "Gene Conflict Resolution" problem is solved using the classic Vertex Cover by constructing a graph G = (V, E) and associating vertices with genes and edges with conflicting pairs. The problem asks if there is a way of finding a minimum size set of genes such that if deleted all inconsistencies will be removed. The solution to the Vertex Cover problem in G will then answer this question. Here we introduce a kernel of size 2k - c for this problem. We, in particular, show that for every positive integer c, there exists a polynomial algorithm that either given an instance (G, k) for the Vertex Cover, constructs another instance (G', k'), where the graph G' contains at most 2k' - c vertices such that  $k' \leq k$  and the graph G has a vertex cover of size k if and only if the graph G' has a vertex cover of size k'. Or the algorithm returns the answer to the Vertex Cover problem. Our result is an improvement over the best known kernel of size 2k introduced by Chen et. al. in 2001.

MSC2000: 05B40,30C10.

Keywords: vertex cover, kernel, FPT.

Session 2

### Encoding Information in a Sudoku Grid

Linzy A. Phillips University of Glamorgan laphilli@glam.ac.uk

(joint work with Stephanie Perkins, Paul A. Roach, Derek H. Smith)

Recently Sudoku, and related structures, have attracted attention for their potential applications to fields such as erasure correction. Sudoku supports the requirements of an erasure-correcting code, as both are concerned with the deduction or recovery of data from partial, given information. One problem that arises is how the information to be transmitted should best be encoded in the Sudoku grid. An initial candidate is the use of the minigrids on the leading diagonal, although it would be desirable to extend this encoding if possible. It can be shown computationally that data suitably written in these minigrids can always be extended to a valid grid, but a theoretical proof would be preferable. In this talk, such a proof is given for the simpler Rudoku puzzle, and extensions are discussed.

MSC2000: 05B30.

Keywords: Sudoku, Rudoku, Erasure-Correcting Codes.

### Unavoidable Sets in Sudoku and Quasi-Magic Sudoku Grids

Sian K. Jones University of Glamorgan skjones@glam.ac.uk

A Sudoku grid is a  $9 \times 9$  Latin Square further subdivided so that nine  $3 \times 3$  'mini-grids' also contain the numbers  $1, \ldots, 9$ . A Quasi-Magic Sudoku has the further constraint that every row, column and diagonal of each mini-grid sums to  $15 \pm 2$ .

A discussion will be given here of how unavoidable sets in Sudoku puzzles and Quasi-Magic Sudoku puzzles affect the uniqueness of their solution.

#### MSC2000: 05B15.

Keywords: Latin Squares, Sudoku, Intercalates.

### Construction of DNA codes

Niema Ali Aboluion Division of Mathematics and Statistics, University of Glamorgan, Pontypridd, CF37 1DL, Wales, U.K. naboluio@glam.ac.uk

Coding theory has several application in genetics. In this talk we concentrate on a specific application from Computational Biology. This concerns the construction of DNA codes using an alphabet of four symbols. These are used in DNA computing, as bar codes in molecular libraries and in microarray technologies. DNA codes constructed satisfy a GC-content constraint and a minimum Hamming distance constraint. They are derived from linear codes over GF(4) and  $Z_4$ , additive codes over GF(4), and their cosets. The software system Magma is used for the implementation. Previous approaches to the construction of these codes are extended in several ways. The codes found include many new bests, and longer codes are constructed than are available in the literature. Future work, which includes the addition of a reverse complement constraint, is outlined.

MSC2000: 94B05, 94B15, 92D20.

Keywords: DNA codes, Linear codes, Cyclic codes.

#### Session 5

### Eleven billion STS(19)s

Tony Forbes Open university anthony.d.forbes@gmail.com

(joint work with Terry Griggs)

A Steiner triple system of order v > 0, also known as an STS(v), is a type of block design consisting of a v-element set of points, V, together with a set,  $\mathcal{B}$ , of 3-element subsets of V called *blocks* (or triples or lines) such that every pair of points appears in precisely one block of  $\mathcal{B}$ . Steiner triple systems exist for every  $v \equiv 1$  or 3 (mod 6) and the size of the block set is v(v-1)/6.

Up to isomorphism the numbers of STS(v)s for v = 1, 3, 7, 9, 13 and 15 are 1, 1, 1, 1, 2 and 80 respectively, and for the block sets of typical representatives of the first four, we have {}, {012}, {012, 034, 135, 236, 146, 245, 056} and {012, 345, 678, 036, 147, 258, 057, 138, 246, 048, 156, 237}. The STS(7) is the familiar projective plane of order 2 and the STS(9) is the affine plane of order 3.

When v = 19 the number is somewhat larger. Last year the STS(19)s were enumerated by Kaski, Östergård, Pottonen & Kiviluto, repeating an earlier computation by Kaski & Östergård in 2000. This time, however, the actual systems were recorded in addition to being counted and are currently available in compressed format to any interested researcher. I received my copy for Christmas 2008, and I shall be talking about some of the things I and others have been doing with this huge finite set of designs. The exact number is 11,084,874,829.

#### MSC2000: 05B07.

Keywords: Steiner triple system.

20 4 Abstracts of Contributed Talks

### Biembeddings of Steiner triple systems using the Bose construction

Costas Psomas The Open University c.psomas@open.ac.uk

(joint work with Prof Jozef Širáň & Prof Terry Griggs)

A face two-colourable triangulation of a complete graph on a surface gives rise to a biembedding of two Steiner triple systems. I will explain how such biembeddings in both orientable and nonorientable surfaces can be constructed starting from the well-known Bose construction.

MSC2000: 05B07, 05C10.

Keywords: Steiner Triple System, Complete Graph, Topological Graph Theory, Embedding.

### 2-Factors in Hamiltonian Graphs

Matthew White Oxford University matthew.white@queens.ox.ac.uk

A well-known theorem of Dirac states that a graph with minimal degree at least  $\frac{n}{2}$  is hamiltonian, this result was extended by Brandt *et al* to show that a graph with minimal degree at least  $\frac{n}{2}$  has a 2-factor with k components for  $4k \leq n$ . Both results are tight, as shown by the complete bipartite graph with classes of order  $\frac{n-1}{2}$  and  $\frac{n+1}{2}$  for n odd. However, this graph has no 2-factors at all, and so a natural question is what minimal degree is required in a hamiltonian graph to guarantee a 2-factor with k components. This talk will look at this problem, with particular empathisis on the case k = 2.

MSC2000: 05C38, 05C45.

Keywords: Hamilton cycle, 2-factor, minimal degree.

Session 7

### Algorithm for Finding k-Vertex Out-trees and its Application to k-Internal Out-branching Problem

EunJung Kim Royal Holloway, University of London eunjung@cs.rhul.ac.uk

(joint work with N. Cohen, F. V. Fomin, G. Gutin, S. Saurabh and A. Yeo)

An *out-tree* is an oriented tree with only one vertex of in-degree zero called the *root*. The k-OUT-TREE problem is the problem of deciding for a given parameter k, whether an input digraph contains a given out-tree with  $k \geq 2$ vertices. In their seminal work on Color Coding Alon, Yuster, and Zwick provided fixed-parameter tractable (FPT) randomized and deterministic algorithms for k-OUT-TREE. While Alon, Yuster, and Zwick only stated that their algorithms are of runtime  $O(2^{O(k)}n)$ , however, it is easy to see that their randomized and deterministic algorithms are of complexity  $O^*((4e)^k)$ and  $O^*(c^k)$ , where  $c \geq 4e$ .

The main results of Alon, Yuster, and Zwick, however, were a new algorithmic approach called Color Coding and a randomized  $O^*((2e)^k)$  algorithm for deciding whether a digraph contains a path with k vertices (the k-PATH problem). Recently Chen et al. and Kneis et al. developed an approach, called Divide-and-Color, that allowed them to design a randomized  $O^*(4^k)$ -time algorithm for k-PATH. Divide-and-Color in Kneis et al. (and essentially in Chen et al.) is 'symmetric', i.e., both colors play similar role and the probability of coloring each vertex in one of the colors is 0.5. We further develop Divideand-Color by making it asymmetric, i.e., the two colors play different roles and the probability of coloring each vertex in one of the colors depends on the color. As a result, we refine the result of Alon, Yuster, and Zwick by obtaining randomized and deterministic algorithms for k-OUT-TREE of runtime  $O^*(5.7^k)$  and  $O^*(5.7^{k+o(k)})$  respectively.

We apply the above deterministic algorithm to obtain a deterministic algorithm of runtime  $O^*(c^k)$ , where c is a constant, for deciding whether an input digraph contains a spanning out-tree with at least k internal vertices. This answers in affirmative a question of Gutin, Razgon and Kim (Proc. AAIM'08).

#### MSC2000: 05C85.

Keywords: subgraph isomorphism, algorithm, tree.

22 4 Abstracts of Contributed Talks

### A Memetic Algorithm for the Multidimensional Assignment Problem

Daniel Karapetyan Royal Holloway, University of London daniel.karapetyan@gmail.com

(joint work with Gregory Gutin)

The Multidimensional Assignment Problem (MAP or s-AP in the case of s dimensions) is an extension of the well-known assignment problem. The most studied case of MAP is 3-AP, though the problems with larger values of s have also a number of applications. In this paper we propose a memetic algorithm for MAP that is a combination of a genetic algorithm with a local search procedure. The main contribution of the paper is an idea of dynamically adjusted generation size that yields an outstanding flexibility of the algorithm to perform well for both small and large fixed running times. To evaluate the effect of the proposed approach, a number of computational experiments for several instance families were conducted.

MSC2000: 90C27, 90C59.

Keywords: Multidimensional Assignment Problem, Metaheuristic, Genetic Algorithm, Memetic Algorithm.

### **Reducing Graphs by Involutions**

John Faben Queen Mary University of London j.faben@maths.qmul.ac.uk

I will give you a graph, and ask you to pick an involution, any involution. Now delete from the original graph all vertices which are not fixed by this involution. You then pick any involution of your new graph, delete all but its fixed points and repeat the process until you have a graph which is involutionfree. Now just by looking at the original graph, I will tell you which graph you are now thinking of.

This is not magic: the choice of involutions, and the order in which they are chosen, is irrelevant. The final graph is uniquely determined by the original. We prove this fact using the theory of confluent reduction systems introduced by Newman in 1942. We also discuss the complexity of determining where this process will end given only the original graph.

#### MSC2000: 05C60,68Q25.

Keywords: graphs, automorphisms, complexity.

# Participants

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