Formal Semantics in Modern Type Theories
(and Event Semantics in MTT-Framework)

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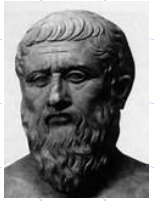
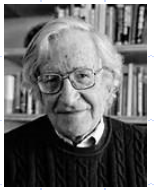
This talk

- I. Formal semantics in Modern Type Theories: overview
  - MTT-semantics is both model-theoretic and proof-theoretic
  - HoTT-logic for MTT-semantics in Martin-Löf's TT
    - paper in Proc. of LACompLing18
- II. Event semantics in MTT-framework
  - (Neo-)Davidsonian event semantics and problems
  - Event semantics in MTT-framework
    - Events in MTT-semantics
    - Event structure with dependent types

I. Overview of MTT-semantics

- Natural Language Semantics – study of meaning (communicate = convey meaning)
- Various kinds of theories of meaning
  - Meaning is reference ("referential theory")
    - Word meanings are things (abstract/concrete) in the world.
    - c.f., Plato, …
  - Meaning is concept ("internalist theory")
    - Word meanings are ideas in the mind.
    - c.f., Aristotle, …, Chomsky.
  - Meaning is use ("use theory")
    - Word meanings are understood by their uses.
    - c.f., Wittgenstein, …, Dummett.





Type-Theoretical Semantics


- Montague Semantics
  - R. Montague (1930–1971)
  - Dominating in linguistic semantics since 1970s
  - Set-theoretic, using simple type theory as intermediate
  - Types ("single-sorted"): e, t, e→t, …
- MTT-semantics: formal semantics in modern type theories

  - Examples of MTTs:
    - Martin-Löf's TT: predicative; non-standard FOL
    - pCIC (Coq) & UTT (Luo 1994): impredicative; HOL
  - Ranta (1994): formal semantics in Martin-Löf's type theory
  - Recent development on MTT-semantics
    - ➔ full-scale alternative to Montague semantics

- Recent development on rich typing in NL semantics
  - Asher, Bekki, Cooper, Grudzińska, Retoré, ...
    - S. Chatzikyriakidis and Z. Luo (eds.) Modern Perspectives in Type Theoretical Sem. Springer, 2017. (Collection on rich typing & …)
  - MTT-semantics is one of these developments.
    - Z. Luo. Formal Semantics in Modern Type Theories with Coercive Subtyping. Linguistics and Philosophy, 35(6). 2012.
    - S. Chatzikyriakidis and Z. Luo. Formal Semantics in Modern Type Theories. Wiley/ISTE. (Monograph on MTT-semantics, to appear)
- Advantages of MTT-semantics, including
  - Both model-theoretic & proof-theoretic – offering a new perspective not available before (explicated later today)

MTT-semantics: basic categories

Category	**Semantic Type**
S	Prop (the type of all propositions)
CNs (book, man, …)	types (each common noun is interpreted as a type)
IV	$A \rightarrow$Prop (A is the "meaningful domain" of a verb)
Adj	$A \rightarrow$Prop (A is the "meaningful domain" of an adjective)
Adv	$\prod A{:}CN.(A \rightarrow Prop) \rightarrow (A \rightarrow Prop)$ (polymorphic on CNs)

In MTT-semantics, CNs are types rather than predicates:

- "man" is interpreted as a type Man : Type.
- Man could be a structured type (say, Σ(Human,male))
- A man talked.
- $\exists m{:}Man.talk(m)$: Prop, where talk : Human→Prop and Man≤Human (subtyping – crucial for MTT-semantics; see later.)

- Rich type structure ("many-sorted", but types have structures):
  - Existing types in MTTs: Table, ∑x:Man.handsome(x), …
  - Newly introduced types to MTTs: Phy•Info (representing copredication)
  - Type-theoretic representations for various linguistic features (Adj/Adv modifications, coordination, copredication, coercions, events, …)
- Selectional restrictions: meaninglessness v.s. falsity

  (#) Tables talk.
  - Montague: ∀x:e.table(x)⊃talk(x) (well-typed, false in the intended model)
  - MTT-sem: ∀x:Table.talk(x) (ill-typed as talk:Human→Prop; meaningless)

  Note:
  - Well-typedness corresponds to meaningfulness (c.f., [Asher11] and others)
  - Typing in MTTs is decidable, while truth/falsity of a formula is not.

Modelling Adjective Modification: Case Study

[Chatzikyriakidis & Luo: FG13, JoLLI17]

Classical classification	**example**	**Characterisation of** Adj(N)	**MTT-semantics**
intersective	handsome man	N & Adj	$\sum$x:Man.handsome(x)
subsective	large mouse	N (Adj depends on N)	large : ΠA:CN. A→Prop large(mouse) : Mouse→Prop
privative	fake gun	$\neg$N	$G = G_R + G_F$ with $G_R \leq_{inl} G$, $G_F \leq_{inr} G$
non-committal	alleged criminal	nothing implied	$\exists$h:Human. $H_{h,A}(\ldots)$

- $H_{h,A}(\ldots)$ expresses, eg, "h alleges …", for various non-committal adjectives A; it uses the Leibniz equality $=_{Prop}$. [Luo 2018] (*)
- cf, work on hyperintensionality (Cresswell, Lappin, Pollard, …)

Note on Subtyping in MTT-semantics

- Simple example

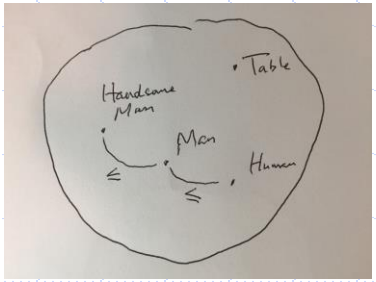
  A human talks. Paul is a handsome man.

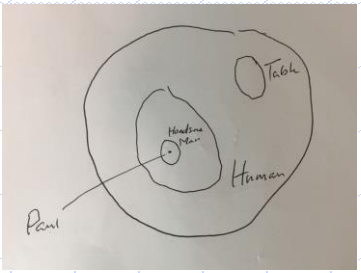
  Does Paul talk?

  Semantically, can we type talk(p)?

  (talk : Human→Prop & p : Σ(Man,handsome))

  Yes, because p : Σ(Man,handsome) ≤ Man ≤ Human.



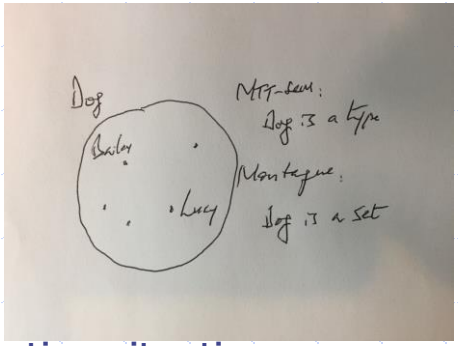


- Subtyping is crucial for MTT-semantics
  - Coercive subtyping [Luo 1999, Luo, Soloviev & Xue 2012] is adequate for MTTs and we use it in MTT-semantics.
  - Note: Traditional subsumptive subtyping is inadequate for MTTs (eg, canonicity fails with subsumption.)

MTT-semantics is both model/proof-theoretic

- Model-theoretic semantics (traditional)
  - Meaning as denotation (Tarski, …)
  - Montague: NL → (simple TT) → set theory
- Proof-theoretic semantics
  - Meaning as inferential use (proof/consequence)
  - Gentzen, Prawitz, Martin-Löf (meaning theory)
- MTT-semantics
  - Both model-theoretic and proof-theoretic – in what sense?
    - Z. Luo. Formal Semantics in Modern Type Theories: Is It Model-theoretic, Proof-theoretic, or Both? Invited talk at LACL14.
  - What does this imply?

- MTT-semantics is model-theoretic
  - NL → MTT (representational/model-theoretic)
  - MTT as meaning-carrying language
    - types representing collections
    - signatures (eg ,subtyping [Lungu 2018]) representing situations
    - Cf, set theory in Montague semantics



- MTT-semantics is proof-theoretic
  - MTTs have proof-theoretic meaning theories
    - Judgements can be understood by means of their inferential roles.
    - Use theory of meaning (Wittgenstein, Dummett, Brandom)
    - Proof-theoretic semantics (Gentzen, Prawitz, Martin-Löf, …)
  - Proof technology: reasoning based on MTT-semantics on computers (eg, [Chatzikyriakidis & Luo (JoLLI14)])

Importance for MTT-semantics

- Model-theoretic – powerful semantic tools
  - Much richer typing mechanisms for formal semantics
  - Powerful contextual mechanism to model situations
- Proof-theoretic – practical reasoning on computers
  - Existing proof technology: proof assistants (Coq, Agda, Lego, …)
  - Applications to NL reasoning
- Leading to both of
  - Wide-range modelling as in model-theoretic semantics
  - Effective inference based on proof-theoretic semantics

Remark: new perspective & new possibility not available before!

Advanced features in MTT-semantics: examples

- Copredication
  - Linguistic phenomenon studied by many (Pustejovsky, Asher, Cooper, Retoré, …)
  - Dot-types in MTTs: formal proposal [Luo 2009] (*), implementation [Xue & Luo 2012] and copredication with quantification [Chatzikyriakidis & Luo 2018]
  - Linguistic feature difficult, if not impossible, to find satisfactory treatment in a CNs-as-predicates framework. (For a mereological one, see [Gotham16].)
- Anaphora analysis/resolution via Σ-types
  - [Sundholm 1986, Ranta 1994] in Martin-Löf's type theory
- Linguistic coercions via coercive subtyping [Asher & Luo 2012]
- Several recent developments
  - (today) Event semantics in MTT-framework [Luo & Soloviev (WoLLIC17)]
  - Propositional forms of judgemental interpretations [Xue et al (NLCS18)]
  - CNs as setoids [Chatzikyriakidis & Luo (J paper for Oslo meeting 2018)]
  - (today) HoTT-logic for MTT-sem in Martin-Löf's TT (current proceedings)

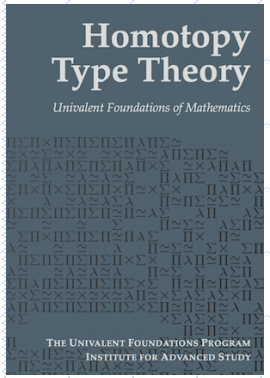
MTT-semantics in Martin-Löf's TT with H-logic

- Martin-Löf's type theory for formal semantics
  - Sundholm, Ranta & many others (all use PaT logic)
- PaT logic: propositions as types (Curry-Howard)
  - P is true if, and only if, p : P for some p.
  - But Martin-Löf goes one step further: types = propositions!
  - This is where a problem arises [Luo (LACL 2012)].
- Proof irrelevance (*)
  - Example: a handsome man is (m,p) : Σx:Man.handsome(x)
    - Two handsome men are the same iff they are the same man – proof irrelevance (any two proofs of the same proposition are the same.)
  - But in MLTT with PaT logic, this would mean every type collapses! Obviously, that would be absurd.
- So, MLTT with PaT logic is actually inadequate for MTT-sem, which has been mainly developed in UTT so far.

$MLTT_h$: Extension of MLTT with H-logic

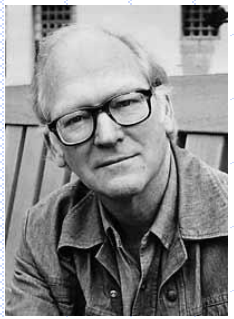
- H-logic (in Homotopy Type Theory; HoTT book)
  - A proposition is a type with at most one object.
  - isProp(A) = $\prod x,y:A.(x=y)$.
  - Logical operators (examples):
    - $P \supset Q = P \to Q$ and $\forall x:A.P = \prod x:A.P$
    - $P \vee Q = |P+Q|$ and $\exists x:A.P = |\Sigma x:A.P|$

    where $|A|$ is propositional truncation, a proper extension.



- $MLTT_h$ = MLTT + h-logic
  - Proof irrelevance is "built-in" in h-logic (by definition).
  - Claim: $MLTT_h$ is adequate for MTT-semantics.
  - Details in the short paper of LACompLing18 proceedings.

II. Events in MTT-Semantic Framework



- Davidson's event semantics [1967]
- Original motivation: adverbial modifications (*)

  (1) John buttered the toast.

  (2) John buttered the toast with the knife in the kitchen.

  Does (2) imply (1)? (Cumbersome in MG with meaning postulates.)
- Events make it natural without meaning postulates.

  In neo-Davidsonian notation with thematic roles (1980s):

  (1') $\exists$e:Event. butter(e)
  & agent(e)=john & patient(e)=toast

  (2') $\exists$e:Event. butter(e) & with(e,knife) & at(e,kitchen)
  & agent(e)=john & patient(e)=toast

  Obviously, (2') $\Rightarrow$ (1')

Problems in Event-semantics + Montague

- For example, "event quantification problem" (EQP)
- Incompatibility between event semantics and MG.

(1) Nobody talked.

Intended neo-Davidsonian event semantics is (2):

(2) $\neg\exists x{:}\mathbf{e}.\ \text{human}(x)\ \&\ \exists v{:}\text{Event}.\ \text{talk}(v)\ \&\ \text{agent}(v,x)$

But the incorrect semantics (3) is also possible – it is well-typed:

(3) $\exists v{:}\text{Event}.\ \neg\exists x{:}\mathbf{e}.\ \text{human}(x)\ \&\ \text{talk}(v)\ \&\ \text{agent}(v,x)$

which moves the event quantifier "$\exists v{:}\text{Event}$" in (2) to the left.

Some proposed solutions to EQP

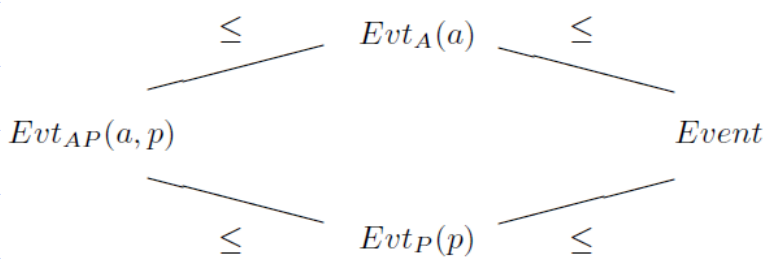
- Many different proposals
  - Purpose: to force scope of event quantifier to be lower.
  - Only mention two of them here.
- Champollion's quantificational event sem. [2010, 2015]
  - talk : (Event→**t**)→**t** with talk(E) = ∃e:Event. e∈E & **talk**(e)
  - Trick: taking a set E of events as argument, but **talk**(e) …
  - Debatable: intuitive meanings, compositionality & complexity
- Winter-Zwarts [2011] & de Groote [2014]
  - Use Abstract Categorial Grammar (see, eg, [de Groote 01])
  - ACG structure prevents incorrect interpretation.
- Our proposal: dependent event types (solution to EQP & …)

Dependent event types [Luo & Soloviev (WoLLIC17)]

- DETs: refining event structure by (dependent) typing
- Applications include
  - A solution to EQP
  - Selection restrictions in MTT-event semantics
- Refined types of events: Event ➔ Evt(…)
  - Event types dependent on thematic roles agents/patients
  - For a:Agent and p:Patient, consider DETs

  Event, $Evt_A(a)$, $Evt_P(p)$, $Evt_{AP}(a,p)$
- Subtyping between DETs:

$$\frac{a : A \quad A \leq B}{a : B}$$



DET-solution to EQP

(1) Nobody talked.

Neo-Davidsonian in Montague's setting (repeated):

(2) $\neg\exists x{:}\mathbf{e}.\ \text{human}(x) \ \&\ \exists v{:}\text{Event}.\ \text{talk}(v) \ \&\ \text{agent}(v,x)$

(3) $\exists v{:}\text{Event}.\ \neg\exists x{:}\mathbf{e}.\ \text{human}(x) \ \&\ \text{talk}(v) \ \&\ \text{agent}(v,x)$

The incorrect (3) is well-typed.

Dependent event types in Montague's setting:

(4) $\neg\exists x{:}\mathbf{e}.\ \text{human}(x) \ \&\ \exists v{:}\text{Evt}_A(x).\ \text{talk}(v)$

(#) $\exists v{:}\text{Evt}_A(x).\ \neg\exists x{:}\mathbf{e}.\ \text{human}(x) \ \&\ \text{talk}(v)$

where (#) is ill-typed since the first "x" is outside scope of "$\exists x{:}\mathbf{e}$".

Selectional restrictions

- Recall:

  (#) Tables talk.

  - Montague: $\forall x{:}\mathbf{e}.\text{talk}(x)$ – well-typed but false, as talk : $\mathbf{e}\rightarrow\mathbf{t}$
  - MTT-sem: $\forall x{:}\text{Table}.\text{talk}(x)$ – ill-typed as talk : Human$\rightarrow$Prop

- What happens with events?
  - Neo-Davidsonian: talk : Event$\rightarrow\mathbf{t}$ or talk : Event$\rightarrow$Prop
  - Montague: $\forall x{:}\mathbf{e}\ \exists v{:}\text{Event}.\ \text{talk}(v)\ \&\ \text{agent}(v)=x$ (well-typed)
  - MTT-sem: $\forall x{:}\text{Table}\ \exists v{:}\text{Evt}_A(x).\ \text{talk}(v)$

    (Also well-typed (!) because Table $\leq$ Agent)

  So?

- Three ways to enforce selectional restriction with events:
  1. Refined typing for verb phrases (like talk)
  2. Refining the typing of thematic roles (like agent)
  3. Refining event types (next slide)
- Approach 1 & 2: Instead of the neo-Davidsonian typing talk : Event$\rightarrow$**t**, or agent : Event$\rightarrow$**e**, we consider
  - $talk_h$: Human$\rightarrow$Event$\rightarrow$Prop (Davidson's original proposal) or
  - $talk_d$: $\prod$h:Human. Evt_A(h)$\rightarrow$Prop (dependent typing) or
  - $agent_h$: Event$\rightarrow$Human (with codomain being Human)
    - Tables talk. (Ill-typed – table x is not a human.)
    - (#) $\forall$x:Table $\exists$v:Event. $talk_h$(x,v) & agent(v)=x (ill-typed)
    - (#) $\forall$x:Table $\exists$v:Event. talk(v) & $agent_h$(v)=x (ill-typed)
    - (#) $\forall$x:Table $\exists$v:Evt_A(x). $talk_d$(x,v) (ill-typed)

- Approach 3: refined DETs
  - Let $T \leq_c$ Agent. (example for subtypes of Agent)
    - $Evt_A[T] : T \rightarrow Type$
    - $Evt_A[T](a) = Evt_A(c(a))$, for any a : T.
- Examples
  - Men talk. (OK because Man≤Human)
  - $\forall x{:}Man\ \exists v{:}Evt_A[Human](x).\ talk(v)$
  - Tables talk. ($Evt_A[Human](x)$ ill-typed as x is not a human.)
  - (#) $\forall x{:}Table\ \exists v{:}Evt_A[Human](x).\ talk(v)$
  - John picked up and mastered the book. (b:Book≤ Phy•Info)
  - $\exists v{:}Evt_{AP}[Human,Phy•Info](j,b).$ pick-up(v) & master(v)

Underlying formal systems

- Systems extended with dependent event types
  - C_e – Church's simple type theory + DETs (with subsumptive subtyping)
  - UTT[E] – the modern type theory UTT + DETs (with coercive subtyping as specified in E)
- Theorem.
  - C_e (like UTT[E]) has nice meta-theoretic properties including, e.g., normalisation and logical consistency.
  - Proof. Faithfully embedding C_e into UTT[E].

(***)

Related (and some future) work on DETs

- Original idea
  - Came from my treatment of an example in (Asher & Luo 12)
    - Evt(h) to represent collection of events conducted by h : Human.
  - Further prompted by de Groote's talk at LENLS14 (on EQP etc.)
- Other applications of DETs
  - For example, problem with negation in event semantics
    - Krifka's solution [1989]: a mereological negation system
    - Champollion's solution [2015] (as mentioned above)
    - DETs solution: details to be worked out.
- DEPs dependent on other parameters
  - Dependency on other thematic roles, say time/location/…: Reasonable? Useful?
  - Dependency on other kinds of parameters than thematic roles?

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