

# Advanced Topics in Formal Semantics Based on Modern Type Theories

(Proposal for Advanced Course in Language and Logic)

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## Abstract

Formal semantics based on modern type theories (MTT-semantics) provides us with not only a viable alternative to Montague's semantics, but potentially an attractive full-blown semantic tool with advantages in many respects. However, the salient features of MTT-semantics may only be discussed properly when more advanced semantic topics are considered. For example, people may ask: how does it fare with semantic issues such as events and anaphoric reference, and could it offer a good treatment of intriguing linguistic features such as copredication? In the past, some of these advanced issues have only been dealt with briefly [9], but recently they have been studied more in depth in the proposer's monograph [33]. In this course, after an overview introduction, I shall consider the following topics in MTT-semantics: event semantics, anaphoric references, copredication and dependent categorial grammars. I'll also compare the treatments with those studied in the Montagovian approach and discuss their respective merits.

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# 1 Motivation and Description

Montagovian formal semantics as currently practised in linguistics is based on Church’s simple type theory and its set-theoretic model theory [11, 19]. During the years, many researchers have realised the benefits of using richer typing systems in formal semantics and proposed various approaches as alternatives or enrichments to the Montagovian approach including [47, 2, 12, 48, 21] and a collection of related papers in [7]. A typical approach is to consider formal semantics in Modern Type Theories<sup>1</sup> (MTT-semantics for short). This work started in the 80s (cf., work by Mönnich [39], Sundholm [50] and Ranta [47]) and many interesting developments MTT-semantics have been made throughout the years [5, 15, 25, 28, 4, 10] and MTTs have now become a viable alternative for the foundational semantic language (see the recent monographs [9, 33]).

However, the salient features of MTT-semantics may only be discussed properly when more advanced semantic topics are considered. For example, people may ask: how does it fare with semantic issues such as events and anaphoric reference, and could it offer a good treatment of intriguing linguistic features such as copredication? In the past, some of these advanced issues have only been dealt with briefly [9] and have not obtained extensive development; recently they have been studied more in depth in the proposer’s monograph [33].

In this course, after an overview introduction, I shall consider the following advanced topics in MTT-semantics: event semantics, anaphoric references, copredication, and dependent categorial grammars. Among these topics include:

- *MTT-event semantics.* Similarly as in Davidson’s event semantics [16], events can be introduced to study MTT-event semantics, as recently done more extensively [33]. In particular, dependent event types (DETs) [35] are very useful in formal semantics. For example, besides their use in solving the Event Quantification Problem [52, 18, 35], the proposer has recently studied how to add the dimension of time in DETs to describe the semantics of tense in language and MTT-event semantics also allows an adequate treatment of selection restriction as well [30, 33].

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<sup>1</sup>By MTTs, we refer to the family of formal systems such as Martin-Löf’s intensional type theory [38, 44], pCIC in Coq [14, 13] and the Unifying Theory of dependent Types [24]. MTTs were initially developed for the foundations of constructive mathematics and later implemented by computer scientists in proof assistants [1, 13, 6] for formalisation of mathematics and verification of programs.

- *MTT-semantic treatment of anaphoric references.* Anaphoric references have motivated the successful studies of dynamic semantics (see [22, 20] among others). Mönnich and Sundholm have applied dependent type theory in the semantic study of donkey sentences [39, 50] where  $\Sigma$  is used to play double roles in representing both the existential quantifier and the subset constructor, and this is problematic [51]. I'll present an improved method in dealing with anaphoric references in MTTs and, as an example, show that donkey sentences can be given satisfactory semantics [32, 33].
- *Substructural Dependent Type Theory.* Categorical grammars have been successfully studied based on Lambek's ordered types [42, 41, 49], Girard's linear types [45, 17, 43, 42, 53], and their combination [23, 40]. The proposer has recently studied a substructural type theory [33, 34] that contains both ordered and linear types (and their dependent versions) and showed, in particular, how dependent types can be applied to study categorical grammars.

We note that, in all of the above topics, new developments have been made recently [33]. Through the discussions, I shall explain how rich type structures in MTTs and the associated powerful mechanisms (for example, polymorphism and fine-grained classifications) can provide useful and adequate tools in semantic constructions. I'll also compare the treatments with those studied in the Montagovian approach and discuss their respective merits.

## 2 Tentative Outline

Tentatively, the course will consist of the following lectures:

- *Monday: Introduction to MTT-semantics.* This introductory lecture will give an overview of MTT-semantics, with basic examples and a case study of adjective modification (see [9], among others). It will lay down enough background about MTTs and MTT-semantics for the lectures in the next days.
- *Tuesday: MTT-event semantics.* After briefly introducing Davidsonian event semantics, I shall explain how events can be introduced into MTTs and explicate that they can not only play the traditional role in formal semantics (e.g., resulting in good adverbial semantics) but make nice contributions by means of dependent event types (DETs)

[35, 36]. I shall illustrate this by means of two case studies: (1) Developing temporal semantics of sense in language by introducing DETs of the form  $Evt_T(t)$ , where  $t : Time$  is a timing argument; (2) Studying *DETs with domains* [30, 33] so that selection restriction can be dealt with nicely by means of (decidable) type checking.

- *Wednesday: Anaphora in MTT-semantics.* In this lecture, I'll first introduce motivating examples about anaphora, using which to illustrate original difficulties and describe the solutions in dynamic semantics [22, 20], and also the previous proposal in dependent type theory and its problems [50, 51]. Then, a new proposal [32, 33] will be presented showing that, in a type theory with both strong  $\Sigma$ -types and weak existential quantifier, satisfactory semantics can be given for the intriguing anaphora that have caused problems in a type-theoretical solution. Discussions will then be made by, for example, considering the non-standardness in logics of dynamic semantics and comparing it with the type-theoretical method.
- *Thursday: Dot-types in MTTs and copredication.* In this lecture, I'll first present examples in copredication [46, 2] and then introduce how the notion of dot-types should be specified in MTTs and applied for copredication in MTT-semantics [25, 28, 8]. Through this discussion, copredication (and its manifestation in more complicated contexts) is used as an example to show that rich type structures in MTTs and the associated mechanisms such as polymorphism and subtyping are powerful in semantic constructions.
- *Friday: Substructural dependent types and categorial grammar.* In this final lecture, I will explain how dependent types can be usefully employed in categorial grammars. Based on our research on dependent substructural type systems [29, 37], a new substructural type theory has been studied and applied to categorial grammar [33]. It contains both ordered types and linear types and, furthermore, their dependent versions. We shall show that the notion of 'terms as proofs' can be extended to that of 'terms as readings' and hence mingle type theories with categorial grammar and, furthermore, study dependent categorial grammars (a paper about this is in preparation [34]).

Course material, including lecture notes, lecture slides and related papers, will be made available to the students.

### 3 Expected Level and Prerequisites

The proposed is an advanced course in the area of Language and Logic. A preliminary background in logic and formal semantics is useful.

### 4 Other Information

#### 4.1 Proposer

The proposed lecturer, Zhaohui Luo, is full professor in Department of Computer Science, Royal Holloway, University of London. An expert in dependent/modern type theories [24, 33], Luo was the co-ordinator of the EU TYPES project in Framework V (34 universities in Europe), a member of TYPES steering committee from 2000 to 2017, and a member of the management committee of the EU COST research network EUTypes from 2017 to 2021. Luo has published extensively on MTT-semantics [28, 9, 25, 26, 27, 31, 3, 7], including a recent research monograph [33], on which the current course proposal is based.

#### 4.2 Previous ESSLLI Courses

The proposer was a lecturer at several ESSLLI summer schools:

- ESSLLI 2011 in Ljubljana (advanced course with Prof Asher);
- ESSLLI 2014 in Tübingen (advanced course with Prof Chatzikyriakidis);
- ESSLLI 2017 in Toulouse (introductory course on MTTs); and
- ESSLLI 2019 in Latvia (advanced course with Prof Chatzikyriakidis).

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